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Analysis and Model Techniques for Determining Dynamic Behaviour of Civil Structures with Various Damping Systems

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1 INTRODUCTION

Controlling vibrations of structures through the use of damping systems continues to increase as structures become lighter and more slender. This includes buildings, towers, vehicular bridges and pedestrian bridges. As a result, the tools used for design of these supplemental damping systems need to take into account the nature of the applied loads to get a more accurate determination of the way the structure will behave with the damping system.

Wind tunnel testing to determine the wind-induced forces on structures has been common for many years. It is common for significant buildings and towers to have wind tunnel testing performed, beginning with the High-Frequency Force-Balance (HFFB) test and occasionally an aeroelastic wind tunnel test is performed to better determine aeroelastic effects such as aerodynamic damping and vortex-induced phenomena. Detailed knowledge of the nature of the wind-induced forces can be used with advanced analysis techniques to accurately describe the behaviour of a supplemental damping system. In addition to wind-induced forces, it may be necessary to consider seismic forces if the structure is located in a region with seismic activity.

To date, damping systems which have been implemented to reduce vibrations of structures have been primarily designed using frequency domain analysis techniques. Beginning with the work of Den-Hartog (1956), many have produced estimates of effective damping of a damped structure and reduction in motion and amplitudes of the damping system. Computing limitations have limited the ability to perform time-domain analysis of the combined structure and damping system. More recently, software commonly available to structural engineers (e.g. SAP2000) has the ability to perform analyses in the time domain. The use of these software packages for time domain analyses is gaining increasing acceptance.

This paper intends to briefly review currently available analysis methods available for analysing the behaviour of a structure with a supplemental damping system. The limitations and advantages of each analysis tool are given. An advanced technique for investigating the behaviour of a structure with a damping system is presented.

2 REVIEW OF ANALYSIS METHODS

Frequency Domain Analysis

The frequency domain type of analysis is computationally efficient, and is based primarily on assumptions about the nature of the forces. For the tuned mass type of damper a simplification of the structure to a simple two-degree-of-freedom (DOF) system is also typically employed. Tuned mass systems would include the well known Tuned Mass Damper (TMD), Tuned Liquid Column Damper (TLCD), Liquid Column Vibration Absorber (LVCA) and the Tuned Liquid Sloshing Damper (TLSD). For wind induced motion it is common to assume that the spectrum of forces can be approximated by a white-noise spectrum. The equations for a 2 DOF system (DenHartog, 1956) can then be used together with the assumption of white noise input forces to arrive at estimates of the building response. (McNamara 1977, Wiesner 1979, Tanaka 1983, Simui and Scanlan 1986.)

The frequency domain method's principle advantage is that it does not require significant computing power. A good approximation of the effective damping provided by a TMD is readily provided. This allows a number of concepts to be quickly investigated, through the use of parameters such as mass ratio, tuning ratio and TMD damping ratio. Another advantage of the frequency domain analysis is the ability to "envelope" the response. Given that there are variations in the frequency content of the input forces, whether wind or seismic, analysis in the frequency domain can ensure that the full range of possible excitation frequencies can be investigated, not just those present in available time histories. This is especially important for seismic events, where time histories of the "design" seismic event are usually synthesized from a code required design spectrum.

Of the limitations of the frequency domain analysis, the primary one is that non-linear effects cannot be easily investigated. For example, off-the-shelf viscous damping units have a force proportional to velocity to the 2nd power. Linear viscous dampers can be produced, but there are additional advantages from using the standard unit, that being the ability to limit motions of a TMD during extreme events. Essentially, due to the higher damping forces produced due to the higher velocities, the TMD becomes overdamped and the amplitudes during the extreme event are reduced compared with those with a linear TMD. Similar non-linearities exist for the TLCD, LCVA and TSLD types of damping systems, some of which are advantageous in design. In addition to limiting the analysis to linear behaviour, analysis in the frequency domain prevents an investigation of more complex coupled TMD-structure interactions.

For many structures, it is sufficient to investigate the seismic behaviour of a structure in the frequency domain. Most code-type calculations find their basis in the frequency domain method combined with a site-specific response spectrum. For more significant structures, it is necessary to investigate non-linear behaviour of structural members. The limitations of analysis in the frequency domain force one to investigate the non-linear (elasto-plastic) behaviour of a seismically loaded structure in the time domain. In considering the design a viscous damping system to reduce wind-induced motion, the forces generated under the design seismic event must also be determined where the structure is located in a seismically active region. As the viscous damping system may have been optimized for wind with its small deflections, the seismic forces generated by the damper can be excessively large if linear viscous damping units are used. A non-linear viscous damper can be utilized, with force being proportional to velocity to a power lower than unity. The total effect of using such a non-linear viscous damper in a structure cannot be readily determined using frequency domain analysis. Similar difficulties arise when visco-elastic dampers are being considered because the non-linearity of the visco-elastic material can be difficult to model in the frequency domain.

Time Domain Analysis

The direct integration of the equations of motion with a variety of input forces for a number of passive damping system types has been reported by several authors (e.g. McNamara, 1977, Kawaguchi et al., 1992, McNamara et al., 2000). Due to the significant computational effort involved, this analysis has generally been used for seismic analyses as the typical seismic event is of much shorter duration than a design wind event. Where a TMD (or TLCD, LCVA, etc.) is being considered, this analysis has in the past only been performed to validate the theoretical basis of the frequency domain method and to verify the selection of parameters determined using the simpler frequency domain analysis. As a result, parametric time domain analyses of the behaviour of a combined building/damping system are not often performed.

Recent advances in computing power and software has given more flexibility to engineers to perform these analyses. Commercially available software packages allow the user to perform analyses in time domain. McNamara et al. (2000) report using this feature to analyse the behaviour of a 39 storey office tower under both wind and seismic loading conditions. However, as commercial software packages compute the response using the fully modelled structure, the number of parametric investigations that can be performed are often still limited by computing time.

For cases where wind-induced motion is being investigated, typically a simulated wind spectrum is used (Kawaguchi et al., 1992). Although the benefit of using measured wind force time histories (for example from a High-Frequency Force-Balance "HFFB" wind tunnel test) as input in the direct integration is understood, to date an efficient method of utilizing all of this data in an efficient integration algorithm has not been developed.

3 ADVANCES IN STRUCTURAL RESPONSE MODELLING

RWDI has developed an efficient means of integrating the response of a structure in the time domain, to both wind and seismic forces. In summary, the numerical integration procedure combines the efficiency of modal analysis with a direct spatial definition of the parts of the damping system. The specific geometry of the damping system, whether it is a TMD or TLCD (or other damping system) is combined with the modal definition of the structure. In this way, coupling of the modes of vibration, torsional effects, etc. can be directly evaluated in the time domain.

Wind-Induced Responses

In analysing the behaviour of a structure to wind-induced forces it is customary to assume that only the fundamental bending and torsional modes are excited by wind forces due to the nature of the wind-force spectrum. This enables us to reduce the description of the building to only the first 3 fully coupled modes of vibration. If necessary, additional modes can be easily modelled. This is a substantial advantage over performing a time domain analysis in the spatial environment with a finite element analysis (FEA) package, where the structural engineer may have a model with thousands of nodes.

This FEA model is still necessary, however, to generate the coupled modes and frequencies for use in this technique. The development of the equations of motion is as follows:

We begin with the basic form of a harmonic oscillatory system,

$$[M]\frac{d^2}{dt^2}\begin{Bmatrix} x_1 \\ y_1 \\ \vdots \\ y_n \\ z_n \\ h \end{Bmatrix} + [C]\frac{d}{dt}\begin{Bmatrix} x_1 \\ y_1 \\ \vdots \\ y_n \\ z_n \\ h \end{Bmatrix} + [K]\begin{Bmatrix} x_1 \\ y_1 \\ \vdots \\ y_n \\ z_n \\ h \end{Bmatrix} = \begin{Bmatrix} F_{1x} \\ F_{1y} \\ \vdots \\ F_{nx} \\ F_{ny} \\ 0 \end{Bmatrix}$$

where the M, C, and K matrices represent the floor-by-floor mass, damping, and stiffness properties of the building/damper system. The position vectors give the x, y, and z deflections of the first floor and are followed by the second floor etc. Similar to what is generally used in a HFFB analysis, a radius of gyration which is characteristic of the topmost floors of the building is multiplied into the rotational component (z) of building motion, to create unit similarity in the position vector. The value h is a single coordinate of damper deflection, and may represent either one principle axis of a TMD, or the column motion of a TLCD etc. For the sake of further discussion we will consider a TMD. The vector of forcing functions is laid out in the same fashion as the position vector.

In this form, significant difficulty is encountered when attempting to define the entries of the C and K matrices, due to our approximation of the large nodal model as a simple chain structure. Therefore, the next step is to separate the M, C, and K matrices into sums of building- and damper-related matrices as

$$\left(\left[M_{building}\right]+\left[M_{TMD}\right]\right)\frac{d^2}{dt^2}\begin{Bmatrix} x_1 \\ y_1 \\ \vdots \\ y_n \\ z_n \\ h \end{Bmatrix}+\left(\left[C_{building}\right]+\left[C_{TMD}\right]\right)\frac{d}{dt}\begin{Bmatrix} x_1 \\ y_1 \\ \vdots \\ y_n \\ z_n \\ h \end{Bmatrix}+\left(\left[K_{building}\right]+\left[K_{TMD}\right]\right)\begin{Bmatrix} x_1 \\ y_1 \\ \vdots \\ y_n \\ z_n \\ h \end{Bmatrix}=\begin{Bmatrix} F_{1x} \\ F_{1y} \\ \vdots \\ F_{nx} \\ F_{ny} \\ 0 \end{Bmatrix}$$

where all terms coupling the TMD to the building are contained within the subscripted TMD matrices with the TMD properties.

The 3 fundamental eigenvectors of the building (excluding the damping device) are augmented with zeroes in the TMD position, and a new vector is added which is a unit translation of the damping device. It has been found convenient to normalize the magnitude of each column of the eigenvector matrix to unity. We then transform the system into a new coordinate set which is a combination of building modal coordinates and TMD displacement. This is accomplished by the substitution $\vec{x}(t) = \Psi\vec{q}(t)$, where Ψ is the augmented eigenvector matrix, and premultiplication of each term by the transpose of Ψ.

$$\Psi^T\left(\left[M_{building}\right]+\left[M_{TMD}\right]\right)\Psi\begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{h} \end{Bmatrix}+\Psi^T\left(\left[C_{building}\right]+\left[C_{TMD}\right]\right)\Psi\begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{h} \end{Bmatrix}+\Psi^T\left(\left[K_{building}\right]+\left[K_{TMD}\right]\right)\Psi\begin{Bmatrix} q_1 \\ q_2 \\ q_3 \\ h \end{Bmatrix}=\Psi^T\begin{Bmatrix} F_{1x} \\ F_{1y} \\ \vdots \\ F_{nx} \\ F_{ny} \\ 0 \end{Bmatrix}$$

The quantities of generalized mass, damping and stiffness are substituted into the building products, and the system of equations becomes:

$$\left(\begin{bmatrix} GM & 0 \\ 0 & 0 \end{bmatrix} + \Psi^T \begin{bmatrix} M & \vec{m} \\ \vec{m} & m \end{bmatrix} \Psi\right)\begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \\ \ddot{q}_3 \\ \ddot{h} \end{Bmatrix} + \left(\begin{bmatrix} GC & 0 \\ 0 & 0 \end{bmatrix} + \Psi^T \begin{bmatrix} C & \vec{c} \\ \vec{c}^T & c \end{bmatrix} \Psi\right)\begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{h} \end{Bmatrix} + \left(\begin{bmatrix} GK & 0 \\ 0 & 0 \end{bmatrix} + \Psi^T \begin{bmatrix} K & \vec{k} \\ \vec{k}^T & k \end{bmatrix} \Psi\right)\begin{Bmatrix} \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \\ \dot{h} \end{Bmatrix} = \begin{Bmatrix} GF \\ 0 \end{Bmatrix}$$

Where GM, GC, and GK are diagonal matrices containing the generalised properties of the building system. The constants m, c, and k are properties of the TMD and are chosen according to design objectives. The submatrices M, C, and K, and vectors $\vec{m}$, $\vec{c}$, and $\vec{k}$, are formulated from a free-body diagram of the building-damper interaction and contain the appropriate coupling terms. To date, we have modelled the interaction of both TMD and TLCD systems, where the appropriate terms are determined assuming that the coordinates of damper device motion and building motion are aligned.

The remaining quantities in this system of equations are the generalised forcing functions due to wind excitation. These time series are available from wind tunnel testing which would have been performed to allow a traditional HFFB analysis. Care must be taken when converting forces measured in the wind tunnel to generalised forces that the eigenvectors are scaled as above. These generalised forces are at model scale, and must be subsequently scaled to full size with the appropriate non-dimensional factors.

In this way, we have simultaneously avoided entry of the unavailable or near-singular building C and K matrices, and reduced the number of equations in the system to a much more manageable size – all without any loss of information or simplifying assumptions.

The versatility of this technique is that it now allows for any type of damping device to be simulated within the building. For example, readily available commercial viscous dampers have velocity squared relationships with respect to force. Such a damper may be easily modelled by the addition of a term that uses velocity times the magnitude of the velocity, while simultaneously retaining the linear velocity dependence of modal damping.

For many software packages, it is necessary to change the above second order system of equations into a set of first order equations, i.e. the state-space representation. Numerical simulation of the above system of equations may then proceed with an accurate algorithm like the 4th order Runge-Kutta scheme. This is a relatively quick process for a typical three hour storm at full scale.

Post processing of the building modal state variables may be performed to yield the spatial response of a desired floor, for example the top occupied floor of the building. Unlike the frequency domain approach, statistical measures of the actual TMD displacement or forces may be evaluated with an Extreme Value type of approach, instead of assuming buffeting behaviour with its attendant peak factor characteristics. Figure 1 shows a sample time history of TMD and top occupied floor responses for a strong across-wind response. The degree to which the TMD resonates with the structure is increased for this scenario, and the effective damping is thus greater. Such a phenomenon is not accounted for by a traditional frequency based approach. Additional data, like peak velocity or power dissipation, assists in the physical specification of components to design the damping system.

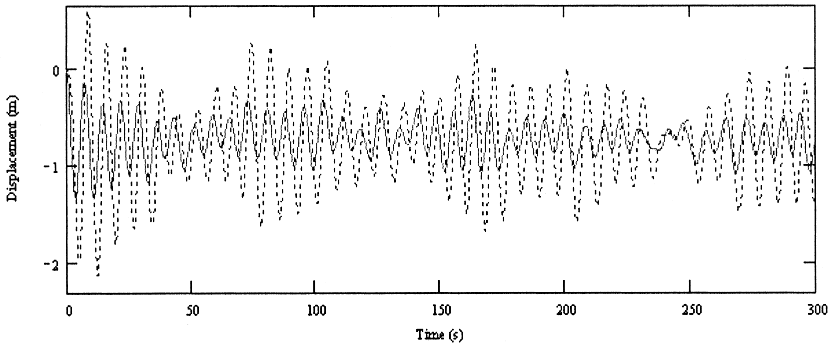

Figure 1 Floor and TMD Motion for an Across-Wind Event.

The numerical simulation may be performed for a range of full scale wind speeds and using data collected in the wind tunnel, can be used to simulate incident wind from multiple sectors around the building. The only change is the input wind tunnel data file, which corresponds to the wind direction. Once all rotational sectors around the building are analysed with the wind tunnel data files, the response of the building has been determined which accounts for the influence of surrounding topography and local structures. Lastly, these results may be combined with meteorological data for the region to determine probabilistic measures of structure and/or TMD response. For example, one can produce within a reasonable degree of effort predicted acceleration levels for a damped structure for a complete range of return periods. This can then be compared to acceleration estimates made by the frequency based HFFB technique to evaluate the effectiveness of a certain set of damping system parameters, such as TMD mass, frequency, damping, etc.

Seismic Responses

When designing a passive damping system to reduce wind-induced motion for a structure in a seismically active region, it may be necessary to evaluate the forces imposed on the structure due to the presence of the damping system. Note that the purpose of the damper is (in our applications) for occupant comfort during wind events, and not to reduce the response of the structure during an earthquake.

The time domain approach proceeds mostly as outlined above. However, it is prudent to make use of a modal model of the building which includes additional modes, either to a reasonable frequency with respect to seismic events for that region, or so as to include most of the mass of the building. In practice, we have found it necessary to include between 10 and 30 modes of the building for a reasonable simulation. Notice that 30 modal coordinates, plus a couple of damper degrees of freedom, still represent a significant computational reduction when compared to the full FE model, yet does not make any substantial sacrifice when macroscopic details are to be investigated.

The second major difference between this and the wind simulation is that of the forcing function. In the wind simulations, all building and damper motion is taken with respect to an inertial frame. For our seismic simulation, without a complete modal model of the building, it is necessary to use a frame of reference that moves with the base of the building. From this frame, we may model excitation forces on each floor by d'Alembert's principle. Our distributed excitation force is then that of ground acceleration, scaled according to the design event, and multiplied by the respective mass of each floor of the building. All post processing of this numerically simulated event must then include a transformation into inertial space, by adding the displacement of the base of the building to the displacement records of all floors of the building.

The principal limitation of this technique is that it simulates the building response, with damping system, to only a single earthquake event, and only from a single incident direction. Many simulations must therefore be run to investigate directional sensitivity, and to attempt to envelope all possible seismic responses.

In an effort to take advantage of the fact that the frequency domain analysis assists with enveloping structural responses, a modification of the National Building Code of Canada (NBCC) frequency domain analysis for seismic events was developed. The NBCC allows for a dynamic analysis to determine the distribution of inter-storey shear loads, which must be scaled to sum to the Code-determined total base shear. The basis of the technique is a summation of modal responses, on a floor-by-floor discretization.

To proceed further in this vein, we must make an assumption. Typically, the structure is assumed to have a modal damping ratio of about 5% against earthquakes in each mode, as per the excitation spectrum in the NBCC. With the addition of a damping device, such as a TMD tuned to the appropriate frequency, we introduce a new mode at a frequency below the fundamental. We make the assumption that the damping ratio of this new mode is also 5%, when in practice it may be 2 to 3 times greater. This is thought to be a conservative assumption, as comparison of damping and restoring forces in the time domain suggests that the restoring forces dominate.

This frequency-based approach with a damping system is begun by first building the system of equations as outlined above for wind, with the exception of the damping matrix. At this point, we are dealing with a system of equations that contains q modal coordinates, and s spatial TMD coordinates for a building with n floors. Solving the new eigenvalue problem produces a set of new natural frequencies and mode shapes for the damped system. We now need to premultiply the second generation eigenvector solution matrix by the original set to create a composite matrix that relates spatial building floor displacements with these new modal coordinates.

The lateral floor loads for each frequency of vibration are determined by

$$P_{ij} = m_i \frac{\sum_{k=1}^{n} m_k \Phi_{kj}}{\sum_{r=1}^{n} m_r \Phi_{rj}^2} \Phi_{ij} \omega_j S_{v(j)}$$

where i represents the building floor, j the response frequency, ω the natural circular frequency of the jth mode, Φ the composite eigenvector matrix, and $S_{v(j)}$ the Code spectral velocity for frequency j. Due to the closely spaced building frequencies at a moderate level of damping, it has been found prudent to combine the modal floor loads with the Complete Quadratic Combination (CQC) technique, as opposed to the simpler Square Root of the Sum of Squares (SRSS) method. Figure 2 illustrates a sample of floor-by-floor shear loads induced by a Code seismic event. Good agreement has been reached with the worst-case time domain simulation, in which numerous local ground acceleration records were applied from multiple directions to the same building.

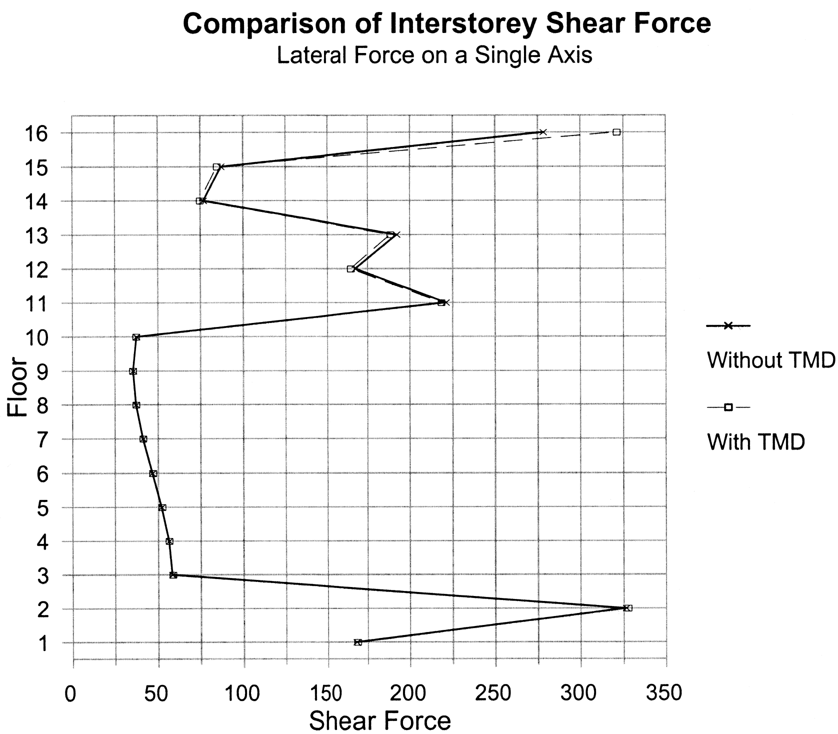

Figure 2 Distribution of Interstorey Shear Forces by Dynamic Analysis.

REFERENCES

Chang, C. C. and Hsu, C. T., 1998
CONTROL PERFORMANCE OF LIQUID COLUMN VIBRATION ABSORBERS, *Engineering Structures*, Vol. 20, No. 7, (Great Britain: Elsevier Science Ltd.) pp. 580–586.

Den Hartog, J. P., 1956
MECHANICAL VIBRATIONS, 4th edition, McGraw-Hill, New York.

Hitchcock, P. A., Kwok, K. C. S., Watkins, R. D. and Samali, B., 1997
CHARACTERISTICS OF LIQUID COLUMN VIBRATION ABSORBERS (LCVA) – I, *Engineering Structures*, Vol. 19, No. 2, (Great Britain: Elsevier Science Ltd.) pp. 126–134.

Kawaguchi, A., Teramura, A. and Omote, Y., 1992
TIME HISTORY RESPONSE OF A TALL BUILDING WITH A TUNED MASS DAMPER UNDER WIND FORCE, *Journal of Wind Engineering and Industrial Aerodynamics*, pp. 1949–1960.

Luft, R. W., 1979
OPTIMAL TUNED MASS DAMPERS FOR BUILDINGS, *Journal of the Structural Division*, ASCE, pp. 2766–2772.

McNamara, R. J., 1977
TUNED MASS DAMPERS FOR BUILDINGS, *Journal of the Structural Division*, ASCE.

McNamara, R. J., Huang, C. D. and Wan, V., 2000
VISCOUS-DAMPER WITH MOTION AMPLIFICATION DEVICE FOR HIGH RISE BUILDING APPLICATIONS, Proceedings ASCE Structures Congress, Philadelphia, May 8–10.

Simiu, E. and Scanlon, R. H., 1986
WIND EFFECTS ON STRUCTURES, 2nd ed., John Wiley, New York.

Tanaka, H. and Mak, C. Y., 1983
EFFECT OF TUNED MASS DAMPERS ON WIND INDUCED RESPONSE OF TALL BUILDINGS, *Journal of Wind Engineering and Industrial Aerodynamics*, pp. 357–368.

Wiesner, K. B., 1979
TUNED MASS DAMPERS TO REDUCE BUILDING MOTION, ASCE Annual Convention and Exposition, Boston, Reprint 3510, April 2–6.