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Effective Stiffness Modeling of Shear Walls for Seismic Analysis of High-Rise Structures

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Introduction

The design of the lateral load resisting systems in high-rise structures is very often governed by the requirements of drift control. This is true even in buildings with relatively rigid lateral systems that include shear walls. Appropriate modeling of the effective stiffness properties of shear walls thus plays an important role in determining the sizes of these shear walls when used, with the accompanying impacts on construction costs, available rentable space, and non-structural components.

Guidance for effective stiffness modeling of shear walls in codes, standards and references while fairly clear as they apply to wind analysis, lack clarity and consistency when it comes to seismic analysis applications.

This paper, based on studies carried out in connection with the recent design of the 41 story cast-in-place concrete St. Regis Museum Tower, believed to be the tallest cast-in-place concrete structure on the West Coast of the USA, examines the recommendations of various available documents that address this issue, and, using a non-linear time-history parametric study based on the structure of the tower, tests appropriateness of the various approaches.

St. Regis Museum Tower

The St. Regis Museum Tower is located in downtown San Francisco, on a site bounded by Mission and Minna Streets on the North and South respectively, and Third St. on the West. The site is approximately 33,000 ft^2 and is generally level. The project consists of a 41story tower above grade; the lower 20 floors occupied by a hotel, the 21st floor mechanical, and the floors above condominiums. The total gross enclosed area of the project is approximately 677,658 ft^2, including four stories of below grade parking. The project also includes the retrofit of an existing nine-story un-reinforced masonry historic building known as the Williams Building. The existing building is tied to and stabilized laterally by the new tower.

The foundation consists of a 6000 psi reinforced concrete mat slab, eight-feet thick at its maximum under the tower core and three-feet thick under low-rise podium areas. Grade 75 reinforcement in one layer was used under the tower to reduce the need for construction steel. The basement walls are generally 18” thick in lower levels, and the basement floor slabs consist of nine-inch thick reinforced concrete flat plates.

The superstructure structural system consists of reinforced concrete framed slabs and flat plates (nine-inch thick flat plates) in the lower podium levels, and eight-inch thick post tensioned flat plates at the typical levels above (nine-inch at the mechanical floor). Core slabs are 12”-thick, conventionally reinforced, and without downturned beams. The lateral system is a “Dual System” consisting of special moment resisting perimeter frames and a shear wall “box” core. The core walls are 2’-0” thick. The moment frame girders at the building perimeter are upturned, to create a flat soffit. Concrete strengths in the lateral system components vary from 5000 psi to 8000 psi typically, with 10,000 psi concrete utilized in specific locations. Figure 1 shows a typical tower floor framing plan.

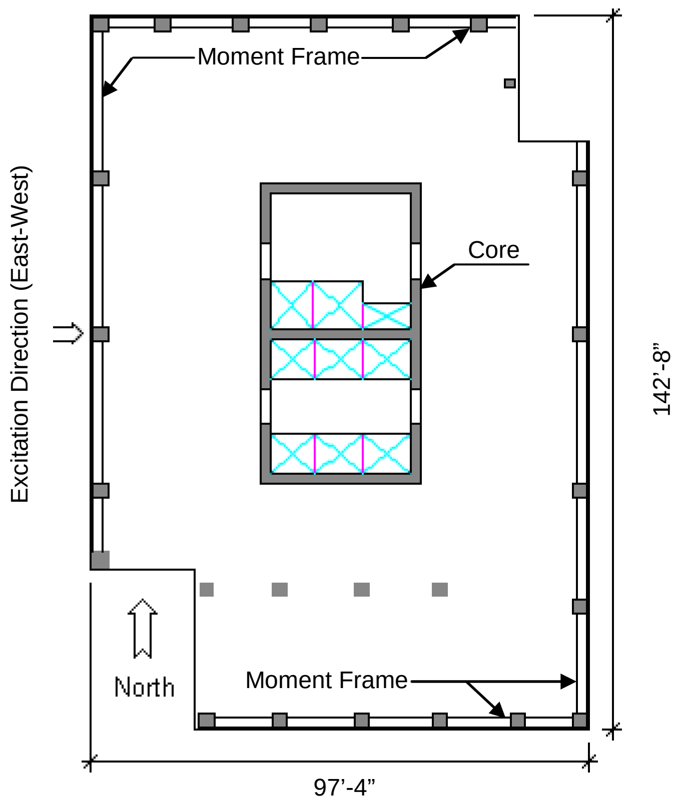


Fig. 1- Typical Tower Floor Plan

The building has a ballroom in the podium whose structure consists of a steel frame, with composite metal deck slabs. The roof of the ballroom has deep long-span steel girders.

The structure of the historic 9-story Williams Building consists of a three-dimensional steel frame with one basement and un-reinforced masonry exterior walls. Its retrofit consists of installing shotcrete shear walls against the exterior masonry walls, a new mat foundation, and new floor diaphragms tied at aligned floors to the corresponding floors of the new tower.

The Issue of Seismic Displacement and "Drift"

Elastic analysis of the structure showed that most of the lateral loads were resisted by the tower's shear wall core, particularly in the lower levels, that code drift limitations governed the sizing of the core wall elements, that the lateral system was "softest" in the east-west direction, and that the podium structure and Williams Building at the base of the tower had a minimal effect on the tower drift.

Design, including drift limitations, was based on the requirements of the 1997 Edition of the Uniform Building Code (UBC). Pursuant to the erratum issued by the International Conference of Building Officials (ICBO) concerning Section 1630.10.3 of the UBC, equation 30-7 was not used to set a lower bound on the seismic force used in determining the building drifts.

In the St. Regis Museum Tower, as is usual in the case of tall buildings whose shear wall cores are primarily responsible for providing lateral stiffness and stability, inter-story drifts, particularly at upper levels are seen to result more from cumulative rotations of the core with height and the corresponding rigid body translations they engender, than from elastic and inelastic deformations within individual stories.

As mentioned earlier, the sizing of the core walls of the St. Regis Museum Tower was governed by the lateral stiffness required to satisfy the code drift limitations, particularly in the east-west direction. Assumptions regarding effective stiffness properties of the structural components, particularly the shear walls, took on paramount importance in order to ensure that drifts were correctly and not over or underestimated. Underestimation of effective stiffness properties could at best result in a soft building with movement perception issues, and the potential of non-structural damage even in relatively minor earthquakes, and overestimation could result in an uneconomical building.

Available Recommendations for Effective Stiffness Modeling

Practical guidance for the selection of effective section properties is available in various codes and references including the UBC, American Concrete Institute ACI 318 (ACI), and Federal Emergency Management Agency FEMA 356 (FEMA) to name a few.

A summary of recommendations for effective stiffness modeling from various codes, and references is presented in Table 1 below. These recommendations are not necessarily consistent with each other and do not necessarily assure that their use will result in optimal designs.

The codes and references listed below all permit the use of alternative effective stiffness properties for components if properly substantiated.

Table 1- Effective Stiffness Modeling

EFFECTIVE STIFFNESS MODELING CODE/REFERENCE APPROACHES			
Code/ Reference	**Element/ Component**	**Flexural Rigidity**	**Shear Rigidity**
OPTION 1 UBC 1997 1633.2.4	All members	0.5 Ec Ig	No Recommend.
OPTION 2 UBC 1997 1910.11	Beams	0.35 Ec Ig	No Recommend.
	Columns	0.7 Ec Ig	
	Walls (uncr.)	0.7 Ec Ig	
	Walls (cracked)	0.35 Ec Ig	
OPTION 3 FEMA 356 6.4.1.2	Beams	0.5 Ec Ig	0.4 Ec Aw
	Columns	0.5 Ec Ig	0.4 Ec Aw
	Walls (uncr.)	0.8 Ec Ig	0.4 Ec Aw
	Walls (cracked)	0.5 Ec Ig	0.4 Ec Aw

Note: For shear stiffness, the quantity 0.4Ec has been used to represent the shear modulus G.

Effective Stiffness Modeling for the St. Regis Museum Tower

For the St. Regis Museum Tower, considering its importance in controlling drift and the potential economic impact of its overestimation, a rational approach was taken in determining the effective stiffness properties of the shear wall core. The effective stiffness was determined using equation from UBC section 1909.5.2.3 repeated here below:

$$Ieff = \left(\frac{Mcr}{Ma}\right)^3 \cdot Ig + \left[1 - \left(\frac{Mcr}{Ma}\right)^3\right] \cdot Icr \qquad \text{(Eq. 1)}$$

Where;

$$Mcr = \frac{fr \cdot Ig}{yt} \qquad \text{(Eq. 2)}$$

and for normal weight concrete

$$fr = 7.5 \cdot \sqrt{f'_c} \qquad \text{(Eq. 3)}$$

Ieff = Effective moment of inertia
Ig = Gross moment of inertia
Icr = Cracked moment of inertia
Ma = Maximum moment for stage at which deflections are being considered
Mcr= Cracking moment
yt = Distance from centroid of gross cross section to extreme fiber in tension

This equation can be rearranged as follows:

$$Ieff = \left\{\left(\frac{Mcr}{Ma}\right)^3 + \left[1 - \left(\frac{Mcr}{Ma}\right)^3\right] \cdot \frac{Icr}{Ig}\right\} \cdot Ig \qquad \text{(Eq. 4)}$$

Considering the critical, east-west direction behavior of the tower (the only direction that will be discussed hereinafter) on account of the essentially solid webs, the three dimensional behavior of the core and the contribution of the “flanges” was taken into account and the value of Ig and Mcr computed for the core as a three dimensional whole. The ratio of Icr/Ig was determined by computing the ratio of the slopes of the moment – curvature graph in the cracked and un-cracked regions of a moment curvature diagram developed for the core cross section using the “UCFyber” program.
(i.e.: using the relationships $M/EI=\phi$ or $M/\phi \propto I$). Ig and Icr can alternatively be directly computed.

The appropriate values of Ma and Mcr to be used were the subject of considerable deliberation. Literal reading of the code would suggest that Ma be the maximum moment induced in the core (integrated over the entire “box” whose effective section properties are being determined) by the code design level earthquake forces determined in accordance with the requirements of UBC section 1630.2. Commonly used analysis programs such as ETABS readily perform this integration. Mcr, similarly, would literally be the cracking moment for the entire core "box" whose effective section properties are being determined.

An alternative approach would be to consider Ma to be My (an approach suggested in FEMA 356) the yield moment capacity of core "box", and Mcr to be the corrected to take into account the beneficial effects of axial load on the core. My, which can be significantly greater than the actual maximum moment induced in the core by code design level earthquake forces. My, can readily be determined using programs such as UCFyber or RC-Section, and the corrected Mcr can determined by revising "fr_{eff}" as follows:

$$fr_{eff} = 7.5 \cdot \sqrt{f'_c} + P_u / A_g \qquad \text{(Eq. 5)}$$

Where P_u is the minimum axial load (0.9 x Dead – Seismic) in the core "box", and A_g is its corresponding area.

Effective section properties for the core "box" determined using the two approaches described above are summarized in Table 2 below. Other approaches and combinations lying between these bounds are also possible but were not studied.

Table 2- Effective Stiffness Modeling

EFFECTIVE STIFFNESS MODELING RATIONAL APPROACHES			
Approaches	**Element**	**Flexural Rigidity**	**Shear Rigidity**
OPTION 4 Ma in core "box"= My	Beams Columns Core Walls	0.5 Ec Ig 0.5 Ec Ig 0.45EcIg (Ma=My)	0.4 Ec Aw 0.4 Ec Aw 0.4 Ec Aw
OPTION 5 Ma in core "box" = actual Max. Code EQ Moment	Beams Columns Core Walls	0.5 Ec Ig 0.5 Ec Ig 0.8 Ec Ig (Ma=Mmax)	0.4 Ec Aw 0.4 Ec Aw 0.4 Ec Aw

The Option 4 approach listed above was the one finally and conservatively selected for use on the project.

A similar approach can also be taken when to model the effective section properties of planar or compound ("I", "T", "L", "C", etc.) shaped shear walls, and members such as columns and girders.

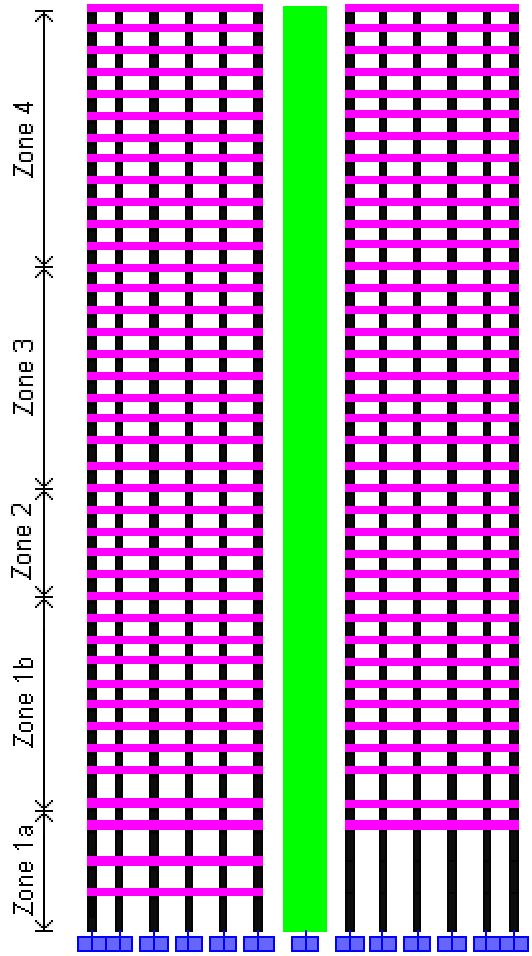
Parametric Study Approach and Model

Given the number of options available for modeling the effective stiffness properties of structural components as described in the preceding section, a decision was made to compare the actual Design Basis Earthquake (DBE) maximum inelastic response displacements determined using non-linear dynamic time-history analysis and accurately determined effective section properties to the corresponding maximum inelastic response displacements computed using the elastic UBC approach. The UBC approach consists in assuming effective member section properties, performing linear elastic time-history analyses at base shears levels scaled to the match the UBC stipulated design level base shear values, determining the elastic design level response displacements "Δ_S" and scaling them up by 0.7 x R (where R is the appropriate UBC stipulated response modification factor) to yield the elastically estimated maximum inelastic response displacements "Δ_M" anticipated in the DBE.

A two-dimensional representation of the structural system for the east-west direction modeled using SAP2000-NL V.8 is depicted in Figure 2. It consisted of the east-west moment frames, and the shear wall core modeled as a single "mega-column" at the center. This was possible, as stated earlier, on account of the relatively solid webs in the east-west direction. The base of the model was fixed at ground level. The minor tapering of the framing at the upper levels and the effects of floor levels below grade were considered insignificant for the purposes of the parametric study and hence not included in the model.

Elements of the moment frames and the core were modeled in five different zones in elevation, matching the variance in sectional geometry, reinforcement and concrete strength in the actual structure. The concrete strength used in the frames and the core varies from 8000 psi to 5000 psi along the height of the building. The moment frames and core were linked with rigid diaphragms at every level.

Mass was assigned to the moment frame girders as uniform line masses, and to the core as lumped masses at ach level using a tributary area approach considering the excitation direction of interest. Gravity loads were included in the model, and “P–delta” effects were considered.



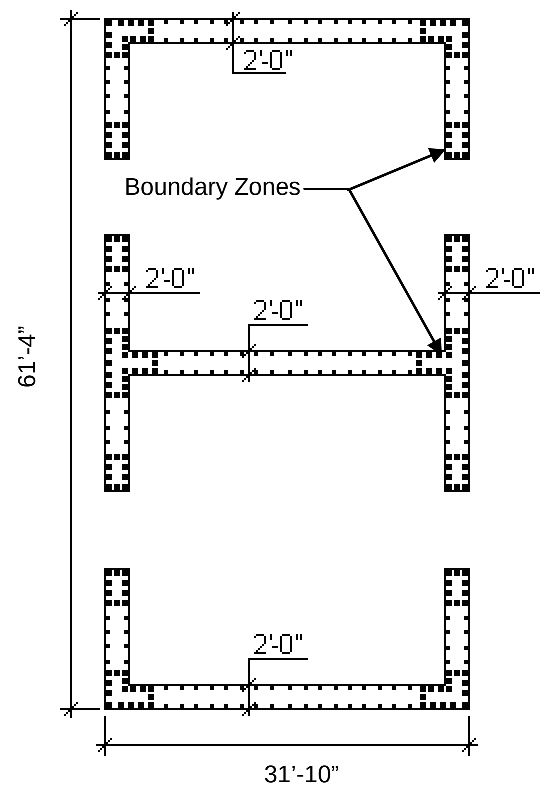
Zones	Beams	Columns	Core Wall
Zone 1a	8000 psi 32"x36"	8000 psi 36"x36"	8000 psi
Zone 1b	8000 psi 29"x30"	8000 psi 36"x36"	8000 psi
Zone 2	7000 psi 29"x30"	7000 psi 36"x36"	7000 psi
Zone 3	6000 psi 29"x30"	6000 psi 36"x36"	6000 psi
Zone 4	5000 psi 25"x30"	5000 psi 36"x36"	5000 psi

Fig. 2- Elevation of Analysis Model

Damping for the analyses, both linear and non-linear, was based on the Rayleigh proportional viscous damping approach. The damping coefficients for the combined mass and stiffness proportional damping were calculated for the first and the tenth modes. Seven percent of critical damping was conservatively used for the non-linear time-history runs.

Plastic Hinge Properties

Material nonlinearities of the structural members were incorporated into the analysis by assigning plastic hinges for each member at each joint.



Note: Horizontal Wall Reinforcement not shown.

Fig. 3- Typical Shear Wall Section used in Moment-Curvature Analysis

The plastic hinge moment-rotation capacity curves were obtained for each zone for beam, column and the shear wall core "mega-column" elements using the "RC-Section" program of the "Pikaso Software Inc."

The structure was sub-divided into five zones with different sectional characteristics for each of these elements (Fig. 2). Each zone represents elements with different concrete strengths, reinforcement and sectional geometry. See Figure 3 for a typical section of the core wall used in the moment curvature analysis to determine the hinge properties of one of the core "mega column" elements.

Moment-curvature relationships for each typical hinge were translated into moment-rotation capacity curves based on plastic hinge lengths of each element. Plastic hinge lengths were typically taken as half the respective member depth. These curves were then matched with the standard bilinear input format for SAP2000 by specifying the control points A, B, C, D and E. See Figure 4 for typical moment-rotation diagrams input into the analysis model (shown superimposed on the corresponding diagrams obtained from the RC-Section program).

Time Histories

Sets of two orthogonal-component time histories selected and scaled per the requirements of UBC Section 1631.6.1 were used as input in the time-history analyses. The three time histories used are listed in Table-3, and depicted in Figure 5. The time histories were spectrally matched, as required by the UBC, to the 5% damped site-specific spectra developed for a DBE earthquake

Table-3. Time Histories used in Analysis

Hazard Level	EQ.	Magnitude	Time-history	Epicent. Dist.(km)	Desig-nation
DBE	Loma	6.9	Corralitos	7	Event 1
DBE	Kocaeli	7.4	Duzce	90	Event 2
DBE	Landers	7.4	Yermo	84	Event 3

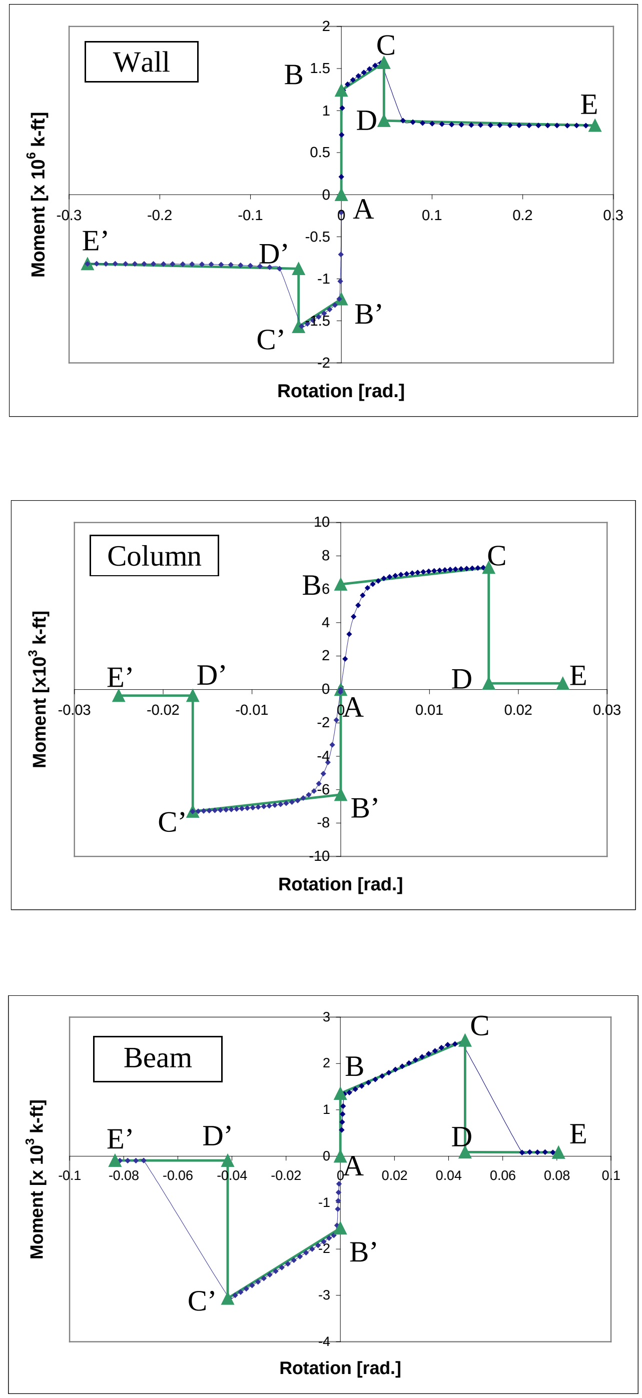


Fig. 4- Typical Moment-Rotation Diagrams

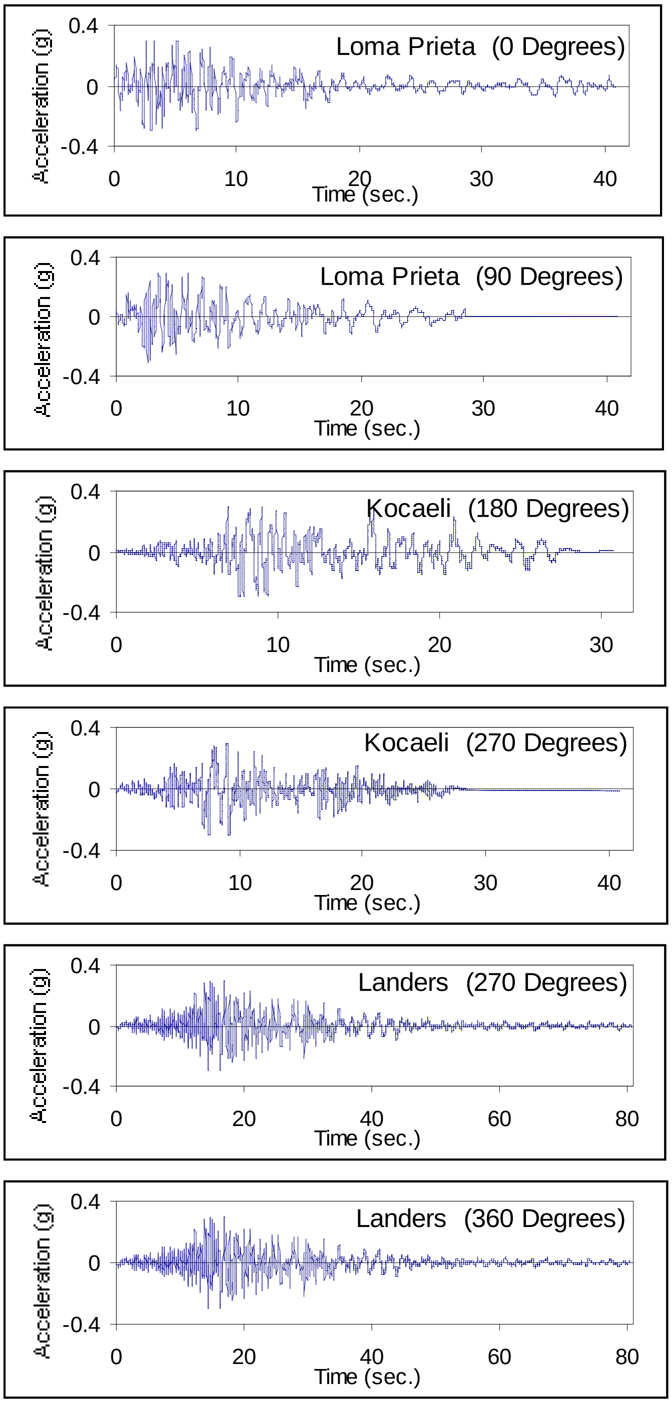


Fig. 5- Acceleration Time Histories

Linear Time-History Analysis

Elastic time-history analyses were performed on the parametric study model for each of the effective stiffness options listed Tables 1 and 2 above and summarized again for convenience in Table 4. For options 2 and 3, the lower two thirds of the core walls were assumed cracked.

The six different time-history inputs were scaled to match the code design level base shear determined per UBC 1630.2 for the drift case (as modified by the corrected Section 1630.10.3). Maximum elastic design level response displacements (Δ_S) obtained were scaled up by a factor equal to 0.7 x R to obtain the estimated inelastic (DBE level) response displacements (Δ_M).

Non-linear Time-history Analysis

Non-linear time-history analyses were then performed on the parametric study model for each of the un-scaled time histories to obtain the actual inelastic response displacements (Δ_M) for each DBE level time- history.

An iterative approach was adopted to estimate the actual effective stiffness for each “zone” of member properties when subjected to the DBE level histories. As an initial step the analysis was performed with columns and beams having a flexural Ieff = 0.5 Ig, and with the shear wall core modeled with Ieff = 0.8 Ig. The results of the first iteration were post-processed to obtain the maximum response moment in selected representative members in each “zone”, and the corresponding Ieff was derived using Eq. 1 (UBC 1909.5.2.3), based on the actual maximum moment “Ma” caused in the representative member in the DBE time-history. The process was repeated till reasonable convergence was achieved. The effective stiffness properties used in the non-linear analyses are summarized in Table 5.

The DBE inelastic response displacements (Δ_M) resulting from both elastic and non-linear time-history analyses are graphically superimposed and summarized for each of the six time histories in Figure 7.

Table - 4. Summary of Effective Stiffness Options Studied

Code/ Guideline	Element/ Component	Flexural Rigidity	Shear Rigidity
OPTION 1 UBC 1997 1633.2.4	All members	0.5 Ec Ig	No Recommend.
OPTION 2 UBC 1997 1910.11	Beams Columns Walls (uncr.) Walls (cracked)	0.35 Ec Ig 0.7 Ec Ig 0.7 Ec Ig 0.35 Ec Ig	No. Recommend.
OPTION 3 FEMA 356 6.4.1.2	Beams Columns Walls (uncr.) Walls (cracked)	0.5 Ec Ig 0.5 Ec Ig 0.8 Ec Ig 0.5 Ec Ig	0.4 Ec Aw 0.4 Ec Aw 0.4 Ec Aw 0.4 Ec Aw
OPTION 4 Ma = My	Beams Columns Core Walls	0.5 Ec Ig 0.5 Ec Ig 0.45EcIg (Ma=My)	0.4 Ec Aw 0.4 Ec Aw 0.4 Ec Aw
OPTION 5 Ma = actual Max. Code EQ Moment	Beams Columns Core Walls	0.5 Ec Ig 0.5 Ec Ig 0.8 Ec Ig (Ma=Mmax)	0.4 Ec Aw 0.4 Ec Aw 0.4 Ec Aw

Comparison of Approaches

Comparison of maximum inelastic DBE response displacements (Δ_M) determined using the UBC approach of scaling up the elastically determined design level response displacements (Δ_S) to the corresponding maximum inelastic response displacements determined directly by non-linear analysis indicates that the linear approaches (options 1 thru 5) yield relatively conservative displacements when compared to the more accurate non-linearly determined inelastic displacements.

It is felt, hence, that in the case of the St. Regis Museum Tower project, there would have been reasonable justification for the use of effective stiffness properties based on the approach of option 5, which employs a rational method based on the use of actual maximum design level response moments to estimate effective member stiffness

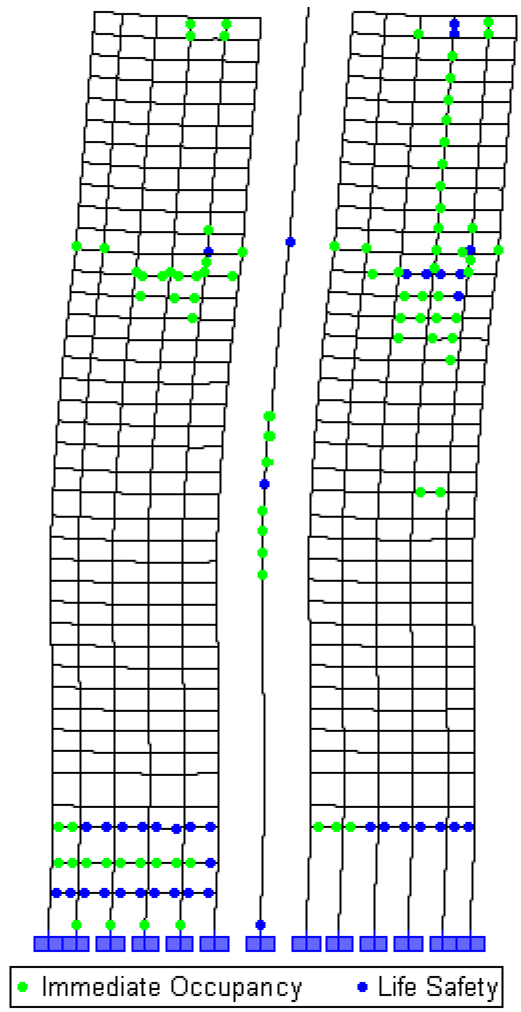


Fig. 6- Hinge Formations Resulting from Non-linear Time-history Analysis
(Analysis results shown for Kocaeli –180 Degrees)

properties. Similar rational approaches could be used on other structures, although judgment would have to be exercised by the designer in selecting a suitable approach that lies within the bounds of options 4 and 5 – for instance considering the beneficial effects of axial loads on the value of Mcr as in the case of option 5 – if a lower degree of conservatism is desired. Use of such an approach could result in larger effective section properties, smaller and less conservative / overestimated drifts and therefore better optimized, efficient and cost effective structures

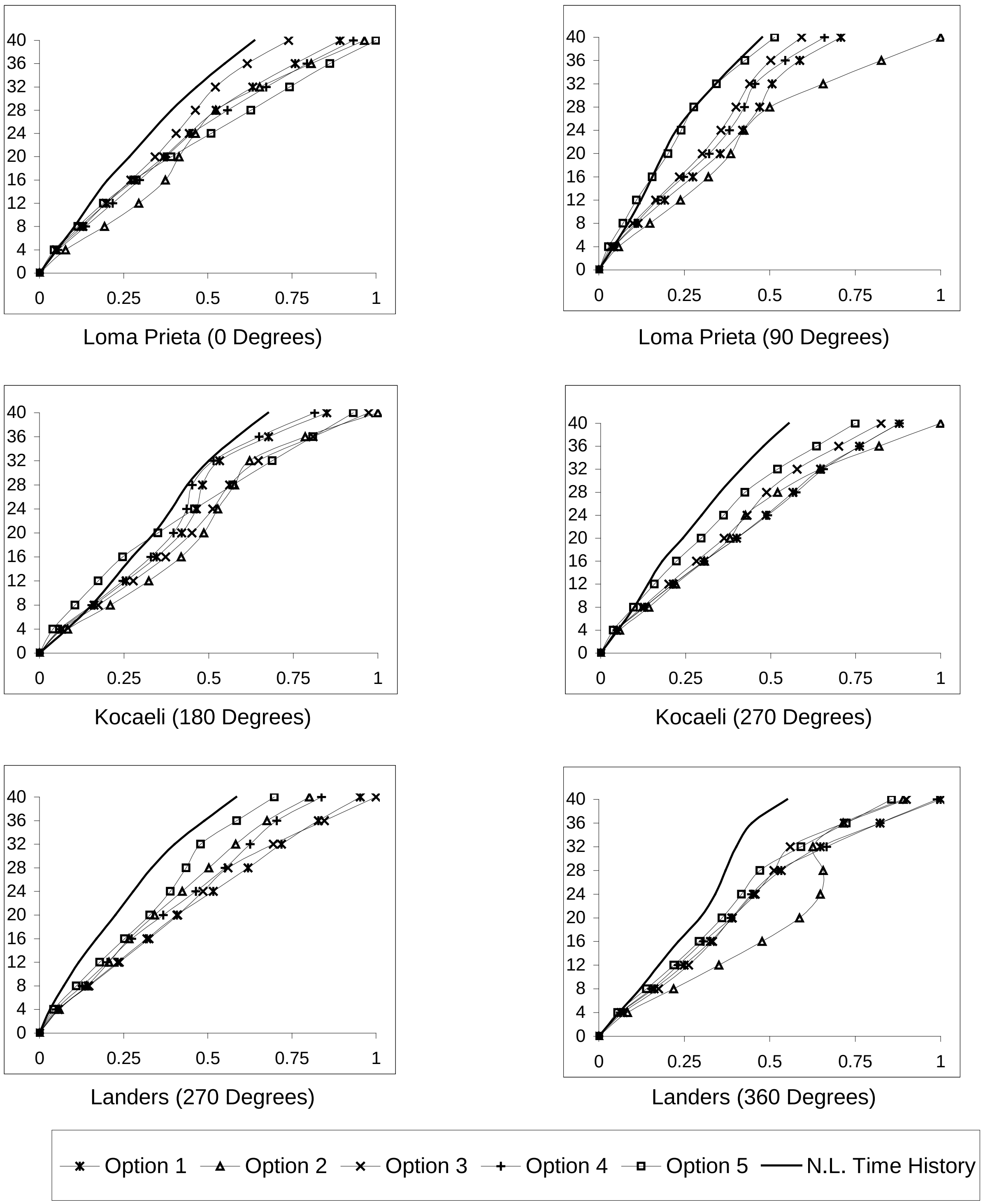


Fig. - 7. Building Inelastic Displacement Envelopes
(Stories vs. Normalized Displacement)

Table- 5. Summary of Effective Stiffness Values based on Non-linear T.H. Analysis

Zones	Beams	Columns		Core Wall
		Exterior	Interior	
Zone 1a	0.19 EI_g	1.0 EI_g	1.0 EI_g	0.54 EI_g
Zone 1b	0.21 EI_g	1.0 EI_g	1.0 EI_g	0.54 EI_g
Zone 2	0.24 EI_g	1.0 EI_g	1.0 EI_g	0.51 EI_g
Zone 3	0.19 EI_g	1.0 EI_g	0.40 EI_g	0.38 EI_g
Zone 4	0.38 EI_g	1.0 EI_g	0.30 EI_g	0.94 EI_g

Conclusion

In drift controlled high-rise structures with lateral systems that include shear walls, seismic displacements are highly sensitive to effective stiffness property assumptions made, particularly for the walls. A number of relatively simple recommendations contained in codes and references, often inconsistent, yield relatively conservative results, causing structures to require needless amounts of costly stiffening.

Economy can be effected by performing non-linear analyses on these buildings to minimize the amount of conservatism, but these analyses can be very complex, time consuming and expensive to perform.

A simpler, rational, elastic approach for estimating effective section properties as described in this paper (option 5) can be judiciously used to more accurately estimate displacements and drifts, reducing over-conservatism and therefore permitting the design of more efficient, better optimized and less expensive structures.

References

ICBO, 1997, *Uniform Building Code, Structural Engineering Provisions*, Vol. 2, 1997 edition, International Conference of Building Officials, Whittier, California.

FEMA 356, 2000, *Prestandard and Commentary for the Seismic Rehabilitation of Buildings*, November 2000, Federal Emergency Management Agency (FEMA), Washington, D.C.

SEAOC, 1999 (Blue Book), *Recommended Lateral Force Requirements and Commentary*, 1999 Seventh Edition, Seismology Committee, Structural Engineers Association of California, California.

ACI 318-99, 1999, *Building Code Requirements for Structural Concrete and Commentary*, 1999 Edition, American Concrete Institute, Farmington Hills, Michigan.

Treadwell & Rollo, Inc., *Ground Motion Time Histories for the St. Regis Museum Tower Project*, January 2001, San Francisco, California.

SAP2000 NL V.8.2.5, 2003, *Structural Analysis Program SAP2000 Nonlinear, Version 8.2.5*, Computers and Structures, Inc, Berkeley, California.

Clough, R. W., Penzien, J., 1993, "Multi Degree of Freedom Systems", *Dynamics of Structures*, 1993 Second Edition.

Park, R., Paulay, T., 1975, *Reinforced Concrete Structures*, New Zealand.