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Authors: Kenichi Ohi, Professor, Kobe University  
Takumi Ito, Research Associate, University of Tokyo  
Zheng-Lin Li, Graduate Student, University of Tokyo

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Sensitivity on Load Carrying Capacity of Frames to Member Disappearance

Kenichi Ohi¹, Takumi Ito², Zheng-lin Li³

¹Professor, Dept. of Architecture & Civil Engineering, Kobe University
²Research Associate, Graduate School of Engineering, University of Tokyo
³Graduate Student, Graduate School of Engineering, University of Tokyo

Abstract
A sensitivity index of vertical load carrying capacity to member disappearance in a framed structures is proposed: The index is defined as the ratio of the load carrying capacity after a member or a set of a few adjacent members disappears to the original load carrying capacity. The member or the set of members that has the highest index may be regarded as a key element or the Achilles heel in the frame. One of the countermeasures against accidental events or attacks is to prepare a careful multiple protection of this Achilles heel. This paper utilizes an efficient linear-programming technique, which is termed as Compact Procedure, to evaluate the load carrying capacities before and after the member disappearance. Planar and 3D frame models are studied about the sensitivity index on vertical load carrying capacity to member disappearance. From these studies, basic properties can be seen on structural sensitivity to member disappearance: (1) In case of strong floor-beam system, corner columns have higher importance than central columns; (2) Violation of symmetric column layout usually results in high sensitivity; (3) An outrigger truss installed at top stories in a multi-story frame is effective to reduce the sensitivity to the column disappearance.

Keywords: vertical load-carrying capacity; member disappearance; sensitivity index; key element; framed structure

Introduction
The world was shaken to witness the progressive collapse of World Trade Center (WTC) on September 11, 2001. Some other examples have been recently reported about collapse of building steel structures by gravity as an outcome resulted from accidental action or excitation not prepared in the design process of the structure: a steel beam was completely sheared off in the turbine facilities in a thermal power plant due to falling of crane equipment during the 2001 Atico earthquake in Peru (Photo 1), and also a few school gymnasia roofs fell down by the 1998 unusual snow coverage in Kanto-Koshin district in Japan (Photo 2).

Even if one or a set of structural member in a building disappears suddenly due to this kind of accidental action, the building should stand against vertical gravity loads, dead and live, not resulting in the whole collapse. And then it is beginning to study how to attain such a performance. A close-up of new concepts, such as redundancy and a key element, has been taken from a viewpoint of assuring robustness.
of a structure even if it receives unexpected disturbance. In such an existing code as the British Standard (BS5950, 1990), a design regulation for a component with high importance (key element) is actually incorporated from a viewpoint of preventing progressive collapse to accidental action.

Frangopol et al. (1987), Wada et al. (1989) studied how much the resistance of a structure would remain after the components of the structure were destroyed by accidental action, and compared it with the resistance at the original state. For instance, Frangopol et al. (1987) proposed an index relevant to the redundancy of a structure by the following equation:

\[
R = \frac{L_{\text{intact}}}{L_{\text{damage}}} = \frac{\lambda}{(\lambda - \lambda^* )}
\]

where \( L_{\text{intact}} \) \( \lambda \) (or \( \lambda_o \) in the next section): Load carrying capacity and the corresponding collapse load factor, respectively, of the structure at original state, \( L_{\text{damage}} \) \( \lambda^* \) (or \( \lambda_{\text{damage}} \) in the next section): Load carrying capacity, and the corresponding collapse load factor, respectively, of the structure at damaged state.

The limit analysis using the linear programming technique is utilized in this research to evaluate the residual load carrying capacity of the framed structure to vertical gravity load after a certain member or a set of member disappear suddenly. This paper evaluates the decreasing ratio before and after the member disappearance, and it is regarded as the sensitivity index of load carrying capacity to the member disappearance. That is, the index shows the importance of the member from a standpoint of preventing the vertical load carrying capacity, and the important member corresponding to the highest sensitivity index shall be regarded as the key element in the structure. An easy way to increase the redundancy of a structure or to obtain high robustness is to protect the key element in multiple manners. This paper demonstrates a few examples of such a key element identification analysis on simple framed structures.

**Sensitivity analysis method**

(1) **Matrix method of limit analysis**

This paper makes use of the matrix method termed ‘Compact Procedure’ proposed by Livesley (1976), which is based on the lower-bound theorem in the limit analysis and the linear programming technique in the optimization problem. The compact procedure solves the following problem:

Maximize \( \lambda \) (load factor)  
Subject to \( \lambda \{ P_o \} = \{ \text{Con.} \} \{ M \} \) (equilibrium)  
\( \{ M \} <= M_p \) (plastic condition)

where \( \{ P_o \} \): Nodal load vector non-factored, \( \{ \text{Con.} \} \): Connectivity matrix in the equilibrium equation, \( \{ M \} \): Member force vector, \( M_p \): Plastic resistance corresponding to member force \( M_f \).

(2) **Sensitivity index to member disappearance**

After Frangopol et al. (1987) this paper evaluates a decreasing ratio of the vertical load carrying capacity of the structural system before and after the disappearance of a certain member, and it is regarded as the sensitivity index to the member disappearance, and denoted by \( S.I. \):

\[
\text{Sensitivity Index: } S.I. = (\lambda_o - \lambda_{\text{damage}}) / \lambda_o
\]

The above formula is equivalent to the reciprocal of the index about the redundancy given by Equation (1) introduced by Frangopol et al. (1987). When the vertical load carrying capacity hardly changes even if a certain member disappears, the corresponding sensitivity index becomes very small, i.e. \( S.I. \approx 0 \). Such a member does not control the load carrying capacity of the whole structural system, and would be regarded less important from the standpoint of reserving the load carrying capacity.

On the other hand, when a member with large sensitivity, i.e. \( S.I. \approx 1 \), disappears, a part of the frame or the whole frame would collapse immediately. Thus such a member with high sensitivity index would be regarded as a key element of the structural system.

(3) **Modification of equilibrium equation**

In the case that a certain member in the frame disappears, the constraints represented by Equations (3) and (4) could be modified in the following way: the member force that disappears is just removed from the member force vector, and at the same time the corresponding column of the connectivity matrix is also removed. This can be performed systematically by a computer program.

\[
\lambda_{\text{damage}} = \begin{bmatrix}
P_0 \\
C_{i1} \\
\vdots \\
C_{ni} \\
\end{bmatrix} = \begin{bmatrix}
\begin{array}{cccc|c}
C_{11} & \cdots & 0 & \cdots & C_{1m} \\
\vdots & \ddots & \vdots & \ddots & \vdots \\
0 & \cdots & C_{nm} \end{array} & \begin{array}{c} M_1 \\
M_2 \\
\vdots \\
M_m \end{array} \\
\end{bmatrix}_{\text{Removed}}
\]

Example I: Single-story planar frame

The single-story four-bay frame shown in Fig.1 is studied about the sensitivity to the column member disappearance. As a vertical loading condition, the uniformly distributed load, the intensity of which is denoted by \( w \) (not factored), is applied on the beam, and this distributed load is replaced by a set of concentrated loads, the magnitude of which are \( P_o \) on the center of each beam and the mid-column tops, and 0.5 \( P_o \) on the side-column tops, where \( P_o = 0.5wH \). Two types of member proportion are studied: One is strong-column type, and the other is strong-beam type.

The plastic condition of member is illustrated in Fig.2. No interaction between flexural and axial
plastic resistances is considered herein. The theoretical interaction based on the four-spring approximation for the member cross-section would be as shown in solid lines of Fig.2. In this approximation, the yield axial force \( N_Y \) and the fully-plastic moment \( M_F \) are equal to \( 4p_Y \) and \( 1.5p_ylim \), respectively, where \( p_Y \) and \( d \) denote the yield resistance of each spring and the depth of member cross-section, respectively. If we assume the depth \( d \) is as much as one-eighth of member length, and this leads to the relationship \( N_Y = 64M_F/3H \). This relationship is assumed also in the following analysis, which is performed under no interaction as shown in broken lines of Fig.2. The member flexural and axial failures are considered only simply-plastic, and no buckling is considered.

Fig.3 shows the ultimate vertical load carrying capacity of two frames in their original states. One member column is removed from the frames, and the collapse load factor \( \lambda_{damage} \) and the sensitivity index \( S.I. \) after one column disappearance are shown in Figs.4 and 5.

While the sensitivity index for the strong-column frame reaches as much as about 80 percent, the sensitivity index for the strong-beam frame remains within smaller level, slightly larger the reciprocal of the number of columns. That is, as for the strong-column type, if one column disappears, a crash of a local floor beam will take place to form a collapse mechanism. On the other hand, as for the strong-beam type, the collapse of the whole floor takes place. Therefore, on the strong-beam type frame, the remaining columns resist effectively to falling of the whole floor and they produce the collaboration effect among the columns.

**Example II: 3D frame with a soft or damaged story**

Consider a multi-story 3D frame with a strong floor-beam system as shown in Fig.6. When a certain accidental event takes place at one story and causes softening or damage at the inter-story structure, the first approximation of the 3D frame may be a rigid body model for the upper story portions so far as the floor-beam system is strong enough to be sound and suffering no damage. A single-story 3D framework of 2 by 2 bays, which has a rigid body in the upper part as shown in Fig.7, is examined herein, and the sensitivity analysis to column disappearance is performed. In the equilibrium equation of this frame, only the force balance in the vertical direction and two horizontal directions and the moment balances about the X, Y, and Z-axis are taken into consideration. In addition, rigid and strong floor-beam system is assumed and the beams do not yield but only the columns yield. No interaction is considered among bi-axially flexural and axial resistances. When the plastic portion of the column is approximated by an eight-spring model for a box type cross-section, the actual interaction surface would be as shown in Fig. 8. Similarly to the previous planar case, the plastic resistances are assumed to satisfy the relation \( N_Y = 16M_{py}/L = 16, M_{py}/L (d/L) = 1/6 \).

The cases analyzed about the column disappearance are summarized in Fig.9. One column, two columns, and three columns disappear in these nine cases. The sensitivity results are shown in Fig.10.

From the result of Fig.10, it can be seen that the disappearance of one column does not cause the significant fall of the vertical load carrying capacity of the structure immediately so far as it has as many as nine columns. As shown in the example of planar frame solved in the previous section, the sensitivity to the side column disappearance sometimes becomes very high. Similarly, when two or more columns disappear together with corner or side columns, the fall of load carrying capacity is considerable, in the contrast to the center column disappearance. When disappearance of three columns occurs, it can be seen that the disappearance of side columns leads to the considerable fall of vertical load carrying capacity.
The sensitivity to the disappearance of three mid-columns is smaller than the sensitivity to only two side columns disappearance. Generally, the drop of load carrying capacity could be predicted from the change of column number taking place, so far as the remaining column arrangement is symmetrical. On the other hand, when asymmetrical, the remaining columns can not work effectively, and then the load carrying capacity drops greatly, compared with the amount predicted from the remaining column number.
Fig. 8. Plastic Failure Surface Considered by Multi-spring

Fig. 9. Overview of Analyzed Cases

Fig. 10. Reduced Capacity after Column Disappearance
Table 1 Plastic Resistance of Members

<table>
<thead>
<tr>
<th>Inter-story</th>
<th>Floor</th>
<th>Member</th>
<th>Flexural Resistance</th>
<th>Axial Resistance</th>
</tr>
</thead>
<tbody>
<tr>
<td>-</td>
<td>2 to 4</td>
<td>Girder</td>
<td>$1.2M_0$</td>
<td>$25.6M_0/h$</td>
</tr>
<tr>
<td>1 to 3</td>
<td>-</td>
<td>Column</td>
<td>$1.44M_0$</td>
<td>$30.7M_0/h$</td>
</tr>
<tr>
<td>3</td>
<td>-</td>
<td>Truss</td>
<td>-</td>
<td>$12.2M_0/h$</td>
</tr>
<tr>
<td>-</td>
<td>5 to PHR</td>
<td>Girder</td>
<td>$M_0$</td>
<td>$21.3M_0/h$</td>
</tr>
<tr>
<td>4 to 7</td>
<td>-</td>
<td>Column</td>
<td>$1.2M_0$</td>
<td>$25.6M_0/h$</td>
</tr>
<tr>
<td>4 to 7</td>
<td>-</td>
<td>Truss</td>
<td>-</td>
<td>$12.2M_0/h$</td>
</tr>
</tbody>
</table>

Example III: Multi-story planar frame

Some of tall buildings have an outrigger truss installed in the upper levels. The FEMA report (2002) and some analytical study (Fukuda, et al., 2003) point out that such an outrigger truss installed in the upper levels of WTC, which suffered from the partial loss of structural components, helped the stress re-distribution effect arising over vertical load, and it was able to stand after a while the impact of a passenger plane. This section deals with two 7-story planar frames with and without an outrigger truss as shown in Fig. 11 in order to verify the effectiveness of this kind of truss on the sensitivity of vertical load carrying capacity. It is referred to as the story height $h$ of the frame, and the span length $2h$ (h for the central short span length).

The column over design factor or the beam-to-column moment capacity ratio at each story is taken 1.2 as a weak-beam structure. The plastic moment capacity of beams for the second floor through the forth floor is taken commonly $1.2M_0$, or 1.2 times fully-plastic moment $M_0$ for the fifth floor through the penthouse roof. The relationship $N_Y = 64M_P/s_0$ is also assumed for the plastic resistances, but the interaction between the axial and flexural plastic resistances is ignored. (Ohi, 2001)

Also the buckling of member is not considered. The plastic capacity of the truss member is assumed to have pinned joints at the both-ends, and strong enough that the lateral component of resistance exceeds (three times greater) the story-shear resistance expected in the surrounding moment resisting frame. The plastic resistances of members are summarized in Table 1.

Vertical loads are applied at each column top and each beam center as concentrated loads. The magnitude of vertical loads at the Roof floor and the penthouse roof floor are taken 1.5 $P_o$, that is, 1.5 times of the standard magnitude $P_o$ at other floors. The vertical load carrying capacity at the original state is shown in Fig.12 for two types of frames. It can be seen that the no difference in load carrying capacity is generated whether with or without an outrigger truss at the original state.
In Figs. 12 and 13, the thick line shows axial yielding, and the circle shows a plastic hinge. In the previous examples of this paper, it was shown that there is a tendency that the sensitivity to the side-column disappearance becomes large, and then, the side-column member at the first floor or the second floor is removed. The sensitivity S.I. is calculated and the results are shown in Fig. 13. Moreover, the mid-column member located directly under the truss member. The sensitivity S.I. is also calculated and the results are shown in Fig. 13. In addition, collapse load factors and the corresponding collapse mechanism are united and shown in this figure.

As a result, in the case of a frame with a truss installed in the upper levels, the frame is in the tendency that the sensitivity to the column disappearance becomes smaller than the sensitivity in the frame consisting of only flexural members, beam-columns. Then, a truss of this kind has the effects to raise the redundancy of a moment-resisting frame against accidental actions.

When a side column disappears in a frame without a truss, the upper part of the column which disappeared is pulled down and a collapse mechanism is finally formed in the upper part of the column. On the other hand, in a frame with a truss installed, the truss member resists to the descent of the upper part of the frame, and the abrupt drop of load carrying capacity is prevented.

Also in the case that the column located in the middle of the frame disappears, on the frame
consisting of only flexural members, the upper part of the frame descends considerably at the position of the column which disappeared, and a collapse mechanism is formed. However, in the case of a frame with a truss in the upper levels, the truss has avoided the abrupt drop of the load carrying capacity of the frame by means of the stress re-distribution effect.

**Concluding remarks**

When a certain member in a frame disappears suddenly, the ratio of the decreased amount of the load carrying capacity after the disappearance to the original load carrying capacity was defined as the sensitivity to the member disappearance. A few example of analysis on planar frames and 3D frames were illustrated.

As for the sensitivity evaluation, only by removing the corresponding column to the member forces which disappear from the original connectivity matrix in the equilibrium equation, and then the sensitivity index is easily evaluated by an efficient matrix method of limit analysis based on the linear programming technique.

It is possible to calculate the sensitivity easily and effectively to identify the key element in a structural system. From the case studies made in this paper, the following conclusions could be derived about the redundancy of a framed structure:

1) In the case of a planar frame with strong-column and weak-beam, the sensitivity to the column member disappearance is large, and in the case of strong-beam weak-column type, the sensitivity is relatively small. This implies that a strong-beam or strong-floor system is important to raise the redundancy of a structure. A weak-beam or weak-floor system sometimes tends to cause a local premature collapse to vertical loads.

2) In the case of the 3D frame with as many as nine columns with strong-beam strong-floor system, the disappearance of one column did not cause the sharp drop of the vertical load carrying capacity immediately. However, when two or more columns disappeared including side-columns, the load carrying capacity decreased greatly compared with the case of disappearance of mid-columns.

3) Multi-story planar frames, with and without a truss system in the upper levels, were examined, and such a truss system prevents the descent of upper portions effectively and lower the sensitivity to the column disappearance.

4) The sensitivity analysis technique or key element identification analysis was effective and suitable for verifying the redundancy of a structural system.

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**References**