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1.0 INTRODUCTION

Tuned mass dampers have been used to add damping to structural and mechanical systems for many years and the theory of their design and behaviour when subjected to sinusoidal forcing has been elegantly described by Den Hartog (1956). The optimization of these dampers with linear dashpots and springs and for sinusoidal excitation is well treated by Den Hartog but considerably less attention has been paid to non-linear devices on systems subject to the random loads imposed by the wind acting on tall buildings and other slender structures such as towers and chimneys.

The present paper deals with the behaviour of non-linear T.M.D.'s subject to random Gaussian excitation. Attention is limited to two simple non-linear forms that occur commonly in practice. These two forms are dry-friction or constant force dampers and "velocity squared" or V^2 damping associated with, for example, flow through an orifice or flow in a rough pipe. Although there is some mathematical development, this is not the essential part of the paper. The prime intention of the paper is to examine the more general characteristics of these non-linear dampers with a view to assessing the advantages and disadvantages of each compared to linear T.M.D.'s.

The characteristics of linear T.M.D.'s are reviewed briefly in Section 2 which includes sample design charts for linear systems. Linearization of the simple friction and V^2 T.M.D.'s is treated in Section 3. The characteristics of non-linear as opposed to linear T.M.D.'s is also addressed in Section 3. Section 4 presents some numerical results and designs for non-linear T.M.D.'s for a fictitious building.

2.0 LINEAR T.M.D.'S

A linear T.M.D. is shown schematically in Fig. 1. The values of M, K and C define the primary system ie the building or tower in question while the values of m, k and c define the T.M.D. The value of the primary mass is a function of the mass and mass distribution of the building, the mode shape under consideration and the modal deflection at the point of attachment of the T.M.D.

For a simple translational mode, the value of M is given by

$$M = \int m(z)\phi^2(z)dz/\phi^2(z_a)$$

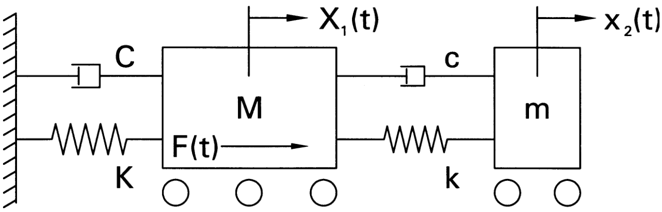

Figure 1 Definition sketch of a S.D. of F. system with an added T.M.D.

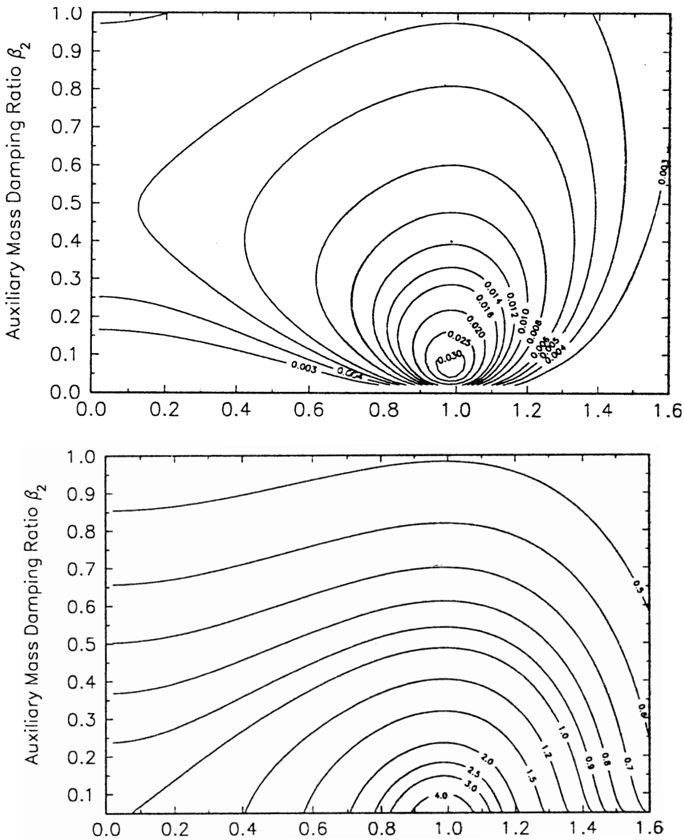

Figure 2 Values of βe (upper) and σ_r/σ_1 (lower) for $\mu = 0.02$, $0 < \beta_2 < 1$ and $0 < f < 1.6$.

where $\phi(z_a)$ = modal deflection at height z
$m(z)$ = mass per unit height
z_a = height of attachment

The value of M is typically about 30% of the actual mass of the building if the point of attachment is at or near the top of the building. The behaviour of the T.M.D. can be expressed in terms of the following dimensionless parameters

μ = mass ratio = m/M; β_2 = damping of T.M.D. = $c/2m\Omega_O$
f = tuning ratio = $\omega\Omega_O/\Omega_O$; g = forcing ratio = ω/Ω_O
β_1 = damping of primary system = $C/2M\Omega_O$

where

Ω_O = $\sqrt{K/M}$; $\omega_O = \sqrt{k/m}$
ω = angular frequency of sinusoidal force acting on the primary system
$F(t)$ = $F_O e^{i\omega t}$

The displacements of the system can be expressed as;

$$X_1(t) = \frac{F_O}{K} H_1(i\omega)e^{i\omega t}$$

$$X_r(t) = X_2(t) - X_1(t) = \frac{F_O}{k} H_r(i\omega)e^{i\omega t}$$

The performance of the T.M.D. is best measured by the amount of damping that it adds to the primary structure. This damping is not easily defined but a satisfactory approach is to define the added damping as that which, when added to the primary system, will result in a response of this S.D.F. system equal to the actual response of the 2 D.O.F. system. If the exciting force is random and the spectrum does not vary strongly in the vicinity of Ω_O then the added damping, β_e, is given as;

$$\beta_e = \left[\frac{4}{\pi} \int_O^\infty |H_1(g)|^2 dg \right]^{-1} - \beta 1$$

β_e is a function of β_1, β_1, μ and f but is only weakly dependent on β_1 and the value of β_e computed with $\beta_1 = 0$ is an adequate approximation. For this special case;

$$|H_1(g)|^2 = [(f^2 - g^2)^2 + (2\beta_2 g)^2]/F(\mu, \beta_2, g)$$
$$|H_R(g)|^2 = g^4/F(\mu, \beta_2, g)$$
$$F(\mu, \beta_2, g) = [(f^2 - g^2)(1 - g^2) - \mu f^2 g^2]^2 + (2\beta_2 g)^2[1 - g^2 - \mu g^2]^2$$

The second parameter that is required is the magnitude of the X_r compared to X_1. The ratio of the rms values (σ_r and σ_1) is given by,

$$\sigma_r/\sigma_1 = [\int |H_r(g)|^2 dg / \int |H_1(g)|^2 dg]^{1/2}$$

The dependency of β_e and σ_r/σ_1 on β_2 and f is shown in Fig. 2 (Vickery et al, 1982) for a mass ratio (μ) of 0.02. The existence of an optimum design to achieve maximum β_e is apparent. The optimum tuning ratio for frequency is slightly lower than unity and the optimum value of β_2 is close to 0.07. In the case of a non-linear damper for which β_2 varies with amplitude, it will be possible only to obtain maximum β_e at some chosen amplitude. The performance for optimum frequency tuning is shown in Fig. 3 for mass ratios of 0.01, 0.02, 0.04 and 0.08.

3.0 LINEARIZATION

Performance diagrams such as those shown in Figs. 2 and 3 can be employed in the designs of linear T.M.D.'s but to employ them for non-linear (friction or V^2) devices requires that the latter be linearized. This can be achieved by defining an equivalent linear β_2 for which the energy dissipation rate matches that of the non-linear device. It is assumed that the non-linearities are such that;

(i) individual cycles are sinusoidal in form with a slowly varying amplitude, $a(t)$
(ii) the distribution of $a(t)$ follows the Rayleigh form associated with a narrow-band Gaussian process.

Both the above assumptions are reasonable and should lead to a linear model which is adequate for preliminary design of a non-linear T.M.D. Final design however should employ a time domain analysis using a true non-linear model of the system that will reproduce the correct probability distribution.

The energy dissipated per cycle of amplitude "a" is given by,

$$E(a) = \pi C_e \omega a^2; \qquad F_D = C_e v$$

$$E(a) = \frac{8}{3} C \omega^2 a^3; \qquad F_D = C v^2$$

$$E(a) = ; \qquad F_D = F_r$$

F_D = damping force

The average dissipated energy per cycle is given by $\overline{E} = \int_0^\infty E(a)p(a)da$; where $p(a)$ is the probability density function defined as $p(a) = a/\sigma^2 \exp(-a^2/2\sigma^2)$.

Evaluation of this integral leads to the following results;

$\overline{E}$ (linear) $= 2\pi c\omega\sigma^2$

$\overline{E}$ (V^2) $= 4\sqrt{2\pi}c\omega^2\sigma^3$

$\overline{E}$ (friction) $= 2\sqrt{2\pi}F_r\sigma)$

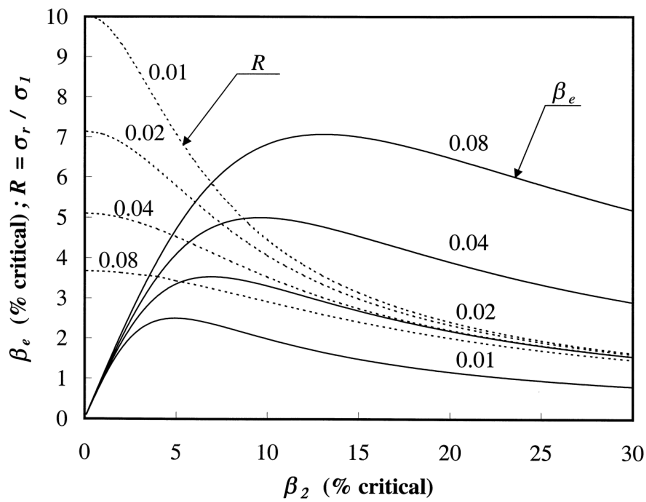

Figure 3 Values of βe and $R = \sigma_r/\sigma_1$ for $f = 1/(1 + \mu)$ and $\mu = 0.01, 0.02, 0.04$ and 0.08.

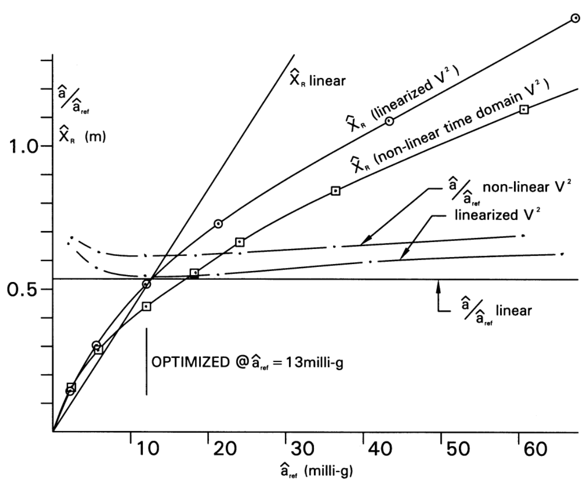

Figure 4 Response of a building with a V^2 T.M.D. with $\mu = 0.094$.

The equivalent linear values of β_2 for the two non-linear devices are then;

$$\beta_2 = \sqrt{\frac{2}{\pi}} \frac{c\sigma_r}{m} \frac{\omega_o}{\Omega_o}; \quad F_D = cV^2$$

$$\beta_2 = \frac{1}{\sqrt{2\pi}} \frac{F_r}{K\sigma_r} \frac{\omega_o}{\Omega_o};$$

σ_r = rms relative displacement , F_r = constant friction force

The above relationships can be used for preliminary sizing of a non-linear T.M.D. The values of β_2 can be used in conjunction with diagrams such as Figs. 2 and 3 but there are some simple relationships that are valid for the small values of the mass ratio ($\mu < 0.10$) likely to be employed in buildings. These relationships are;

(i) for optimum performance

$$f = \frac{1}{1+\mu}, \beta_2 = \sqrt{\mu}/2, \beta_e = \sqrt{\mu}/4, R = \sigma_r/\sigma_1 = 1/\sqrt{2\mu}$$

(ii) for $\beta_2 \to 0; f = \dfrac{1}{1+\mu}; \beta_e = \beta_2, R = 1/\sqrt{\mu}$

(iii) for $\beta_2 \to \propto ; f = \dfrac{1}{1+\mu}; \beta_e = \dfrac{\mu}{4\beta_2}, R = \dfrac{1}{2\beta_2}$

The adequacy of the linearized approach is illustrated in Fig. 4 in which the results of the linearized approach are compared with those obtained from a time-domain non-linear simulation. The results in Fig. 4 are for an actual damper with the following properties;

mass = 250,000 kg ; $\mu = 0.0094$; $F_D = CV^2$; $\beta_o = 0.01; f_o = 0.16$ Hz

The damper is designed to have an optimum performance when the peak acceleration (without the T.M.D.) is 13 milli-g and is expected to reduce the acceleration by at least 35%. For optimum performance we have;

$f = 1/1 + \mu = 0.99$, $\beta_2 = \sqrt{\mu}/2 = 0.0485$, $\beta_e = \sqrt{\mu}/4 = 0.0242$
$R = \sigma_R/\sigma_1 = 1\sqrt{2\mu} = 7.29$

The acceleration of the building is given by; $\hat{a} = 13/\sqrt{1 + \beta_e/\beta_o}$ milli-g

$\hat{x}_1 = 0.128/\sqrt{1 + \beta_e/\beta_o}$ M = 0.069 m

The peak relative motion is $0.069 \times R$ or 0.503m and the root-mean-square value is $0.503/3.6 = 0.140$m. The peak factor of 3.6 is consistent with a Gaussian process with a sample length of 30 minutes. The required value of c is then given by;

$\beta_2 = 0.0485 - \sqrt{\pi/2} \bullet c\sigma_R/m \bullet \omega_o/\Omega_o$; or
$C = 110{,}000\ N/(m/s)^2$

This value of C was confirmed by the true non-linear time domain simulation. Fig. 4 shows the predicted reductions in the peak acceleration as a function of $\hat{a}_{ref}$, the peak acceleration in the absence of the T.M.D. Also shown are the peak displacements for the T.M.D. Results are presented for,

(i) a non-linear ($F_D = CV^2$) system predicted using the linearized theory,
(ii) a non-linear system predicted by time domain simulation; and
(iii) a linear system optimized for $\mu = 0.0094$.

There are a number of features of Fig. 4 that deserve comment.

(i) The V^2 system compares favourably with the optimized linear system for reference accelerations from the perception level to 60 milli-g. The acceleration of 13 milli-g corresponds to a return period of about 10 years while for a return period of 1000 years the reference acceleration is 45 milli-g.
(ii) The linearized predictions of the peak accelerations are about 10% lower than those from the simulation.
(iii) The predicted motions of the T.M.D. given by the linearized model are about 20% higher than those given by the simulation.
(iv) At the 1000-year return period the motions of the T.M.D. in the V^2 system are some 50% lower than those in an optimized linear T.M.D.

4.0 PRELIMINARY DESIGN OF A V² T.M.D.

The building under consideration has a natural frequency of 0.20 Hz, a damping of 1% of critical, a height of 200m, a cross-section of 30m × 30m, a bulk density of 240 kg/m^3 and a mode shape $\phi(z) = (z/H)^{1.23}$. For a 10-year return period, the tip acceleration is 20 milli-g and it is desired to reduce this to a maximum of 12 milli-g. the T.M.D. will be installed at or very near $z = H$. For R = 1000 years, the tip acceleration without additional damping is 70 milli-g and the damper must accommodate this motion.

The chosen design is a simple "U-tube" damper as illustrated in Fig. 5. Dampers of this general type have been discussed by Sakai et al (1989). The aimed reduction in acceleration is from 20 to 12 milli-g but an allowance should be made for inaccuracies in the design method and the inaccuracies of the theoretical and experimental data. Somewhat arbitrarily, the target acceleration will be chosen as 10 milli-g, thus allowing for a 20% error in the design process. For an optimum damper, the added damping is $\sqrt{\mu/4 \text{ or } 0.0354 \text{ for}}$ $\mu = 0.02$, this would reduce the acceleration by a factor of $1/\sqrt{1 + 0.354/.01}$ for or 0.47 (i.e., to 9.4 milli-g) and thus provide an adequate margin on the target value of 12 milli-g.

The exact analysis of a U-tube damper is complex but a simple one-dimensional analysis enables the parameters x_r, m, c and k for the system in Fig. 1 to be evaluated. The results of such an analysis are;

$$m \cong w\,B\,D\,L; \cong (L_e/L)x_o$$

$$c \text{ (where } F_D = c\dot{x}_r^2) = C_L \frac{\rho}{2} A_o (L/L_e)^2$$

$$k = 2(A_o^2/A_R)\,(L/L_e)\,(\gamma + K_a/H) \text{ both reservoirs sealed}$$
$$= 2(A_o^2/A_R)\,(L/L_e)\,(\gamma + K_a/2H) \text{ one reservoir sealed}$$

where; $\dot{x}_o$ = average flow velocity over A_O; γ = specific weight of water; K_a = adiabatic bulk modulus of air; $L_e = \int_1^2 \frac{A_O}{A_S} ds$; A_S = area of stream tube for one-dimensional analysis; C_L = loss coefficient of screen, orifices, etc.

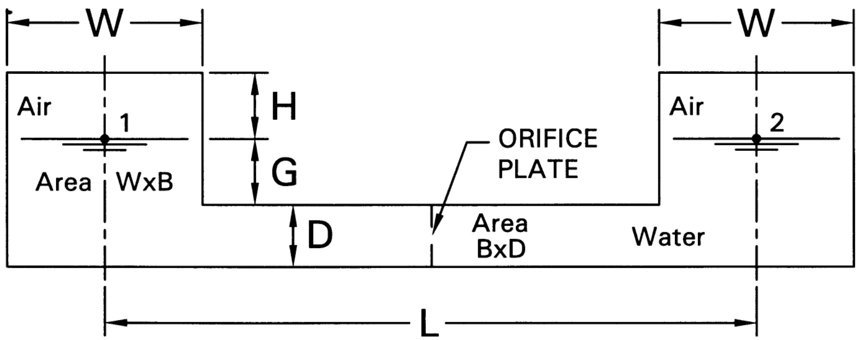

Figure 5 Arrangement of a "U-tube" V^2 T.M.D.

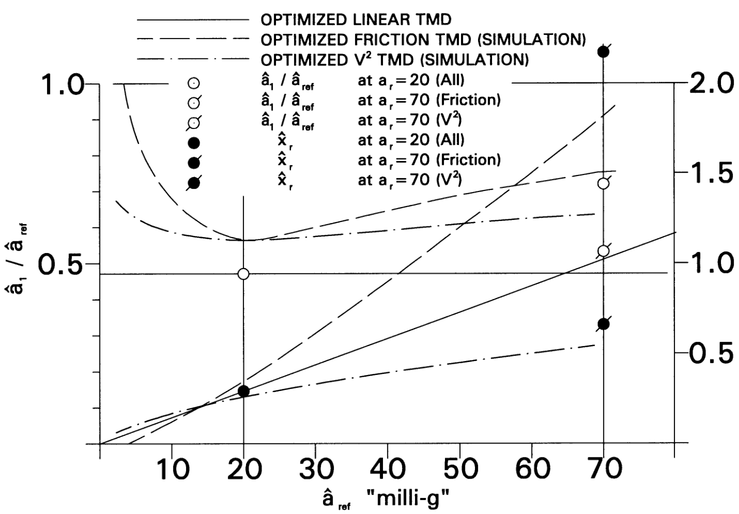

Figure 6 Performance of linear, friction and V^2 T.M.D.'s with $\mu = 0.02$.

For $\mu = 2\%$, $\sigma LBW = 250{,}000$ or $LBW = 250\ \mathrm{m}^3$. The value of β_2 is given by;

$$\beta_2 = \frac{\sqrt{\mu}}{2} = 0.0707 = \sqrt{2/\pi}\,(c\sigma_R/m)(\omega_O/\Omega_O)$$

$$\omega_O/\Omega_O = 1/1 + \mu = 0.98,\ \sigma_R = \sigma_R/R,\ R = \sqrt{2/\pi} = 5$$

The value of σ_1 can be computed from the peak acceleration of 9.4 milli-g; for a peak factor of 3.6 the value of σ_1 is $9.4 \times 9.81/1000/3.6/\Omega_O^2 = 0.016$ and hence $\sigma_R = 5\sigma_1$ 0.081m. Using this value for σ_R the required value of C is 279,000 $\mathrm{N/(m/s)^2}$ and hence $\sum C_L = 62$ for an assumed value of $L/L_e = 0.95$. The value of k must be chosen to yield $f_O = 0.20/1.02 = 1/2\pi\sqrt{k/m}$.

There are of course a variety of designs that will satisfy the requirements but that chosen is as follows;

$D = 1.0\mathrm{m}$, $W = 3.0\mathrm{m}$, $L = 25.0\mathrm{m}$, $B = 10.0\mathrm{m}$,
$H = 0.90\mathrm{m}$, $G = 0.90\mathrm{m}$ one reservoir sealed, one vented

The orifice plate might be a plate with 48mm diameter holes at 100mm centres and supported on a frame to resist the imposed loads. Final tuning of the system after on-site measurements are comparatively simple. The frequency tuning is accomplished by increasing or decreasing H, i.e. by adding or removing water. Damping tuning can be accomplished by blocking off or opening out a selected number of orifices.

Dealing with the 1000-year accelerations is not a problem since the motions of the water are very small compared to the available free space of about 1.0m. Design details will not be covered in any depth but it is worthy of note that the primary source of stiffness is the "air spring" in the sealed reservoir. This spring provides 87% of the system stiffness and hence small surface waves will not markedly influence the performance. Even so it would be prudent to use vertical baffle plates in the end reservoirs to ensure that resonant waves will not develop.

The use of turning vanes to link the tube to the reservoir is also a consideration. These would lead to an even distribution of the flow into the reservoir. The selection of the orifice plate (or alternately screens) is important since the available data is from steady flow rather than oscillating flows. The steady flow condition will be approached if $a >> d$ where a is the amplitude of the fluid motion and d the hole diameter (or bar size in a grid). In the present instance, a typical value of a (2σ say) is 0.16m and the value of a/d is 6.7 which should ensure that the orifice plate performance is similar to that under steady flow conditions but tests would be advisable to examine the performance for a range of a/d. For $R = 1000$ years the computed value of σ_R is 0.18m and the peak value $\hat{x}_R$ is 0.648m. This corresponds to a maximum rise in the reservoir of 0.205m and a maximum flow velocity in the tube of 0.78 m/s. These two values define the design pressure in the sealed reservoir, $\Delta p_{max} = K_a \times 0.205/H =$ 27.5 kPa and the pressure drop across the orifice plate, $p = C_L \rho V^2 = 18.9$ kPa. This could be spread across many orifice plates or screens rather than concentrated at the single orifice plate of this example.

The performance of the above damper is shown in Fig. 6 together with that for an optimized linear T.M.D. and an optimized friction damper with the optimum value of F_r being 5560N for $\mu = 0.02$. The curves shown in Fig. 6 were determined using a non-linear time domain simulation but also shown are the points predicted using the linearized theory. These predictions are shown for $\hat{a}_{ref}$ (the peak acceleration in the system without a T.M.D.) equal to 20 milli-g and 70 milli-g. The linearized theory closely predicts the optimum values of C and F_r and provides reasonable estimates of the reduction in acceleration and the motions of the T.M.D.

A problem with the friction damper are the large T.M.D. motions at excitation levels substantially higher than that at the design point. The amplitude reduction also falls off quite rapidly away from the design point. The friction damper is more suited to situations where the design point corresponds to maximum excitation eg, when used to reduce vortex-induced motions in the vicinity of the critical speed.

5.0 CONCLUSION

The linearization of V^2 and friction T.M.D.'s provides a simple and adequate approach to optimizing design and predicting performance but time domain non-linear simulation should be employed to refine performance predictions.

The V^2 damper significantly reduces motions of the T.M.D. with little sacrifice in performance compared to the optimum linear T.M.D.

The friction damper performs well in the vicinity of the design point but results in large T.M.D. motions at higher levels of excitation.

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