Shear Behavior of R/C Beams with Web Openings Reinforced by Prestress Force

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Abstract
In tall building, there is a trend towards providing openings through the webs of reinforced concrete (R/C) beams to accommodate service ducts. But R/C beams with web openings have sectional loss caused by web opening. So sometimes severe cracks having large width may appear near web opening, and those severe cracks may affect seismic performance of R/C beams.

To improve those situations, the authors have proposed new reinforcing method for web opening of R/C beams. That new method is to introduce prestress force to area around web openings; the force is loaded by prestress tendons arranged near web openings. In this study, the authors performed shear-bending experiments of R/C beam specimens to investigate seismic characteristics of beams having such reinforcing methods.

From experiments the following fact made clear; (1) Strength of specimens when shear crack occurred near web opening grew according to growth of introduced prestress force for tendons, (2) When prestress force was introduced, rack width near web opening could be controlled by introduced prestress force for tendons, and (3) Shear ultimate strength of specimens could be estimated by the formula.

The authors confirmed that the proposed new methods were effective for reinforcing of R/C beam with web opening.

Keywords: Reinforced Concrete, Beam, Web Opening, Prestress, Tendons

1. Introduction
In tall building construction, there is a trend towards providing openings through the webs of reinforced concrete (R/C) beams to accommodate service ducts. R/C beams with web opening have section loss caused by web opening, so their section loss may cause large crack width around web opening. Sometimes their large crack width may exceed beyond design assumption. To improve that situation, the authors has proposed new reinforcing method, named “the IC (inner confinement) reinforcing method; the IC reinforcing method is made up of unbonded-type prestress tendons arranged in inside of beams around web opening and introducing prestress tensile force to prestress tendons. The authors intend that the IC reinforcing method may be adopted for new construction buildings for restraint and control of crack width being small.

In this research the authors intended to investigate effects of amount of prestress tensile force to prestress tendons, ratio of web opening reinforcement by prestress tendons and compressive strength of concrete, for ability of restraint of crack width and shear ultimate strength of reinforced concrete beams having the IC reinforcing method.

The authors, furthermore, have proposed new reinforcing method, named “the OC (outer confinement) reinforcing method”, for beams with web openings in existing building; the OC reinforcing method is made up of prestress tendons placed on the surface of beam roundly, and introducing prestress tensile force to the tendons. In this study the authors also intended to investigate effect of the OC reinforcing method.

2. Experimental Program
Specimen’s details are shown in Fig. 1, and Table 1. Twelve specimens were tested in this study. Eleven of specimens were made as specimens of the IC reinforcing method, and one of specimen was as the OC reinforcing method. All specimens had 300mm in
section width $b$ and 450mm in depth $D$ of section, 150mm in diameter of web opening (equal to $D/3$) and 1.54 of shear span ratio $M/Qd$.

Six pieces of D22 (deformed bar having 22mm in diameter) high strength bar having screw-type knot were used as longitudinal reinforcement. Three pieces of U8 (deformed high strength PC tendons having 8mm in diameter), which were arranged as stirrup in 100mm intervals, and round sectional unbonded-type prestress tendons 7.1Φ, 9.2Φ and 11Φ (having 7.1mm, 9.2mm and 11mm in diameter) were used as reinforcement for web opening.

On the IC reinforcing method specimens (except Spec. No.3), four pieces of unbonded-type prestress tendons were arranged symmetrically having 40 degree of inclined angle against longitudinal direction in inside of specimens.

On the OC reinforcing method specimen (Spec. No.3), a pair of prestress tendons was placed round on the surface of specimen near web opening. Those round tendons were connected each other by steel corner blocks, and same amount of prestressed tensile force was loaded to those tendons, not only tendons placed to beam-depth direction but also beam-width direction.

On all twelve specimens, prestressed tensile forces for tendons were loaded by the post tension-style loading. To normalize the effects of prestressed tensile

<table>
<thead>
<tr>
<th>Spec. No.</th>
<th>PS tendons’ diameter, mm</th>
<th>Reinf. method</th>
<th>$F_t$ MPa</th>
<th>$p_p$ %</th>
<th>$P_t$ kN</th>
<th>$P/P_y$</th>
<th>$\sigma_d$ N/mm²</th>
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IC: IC reinforcing method  OC: OC reinforcing method  $F_c$: Designed concrete compressive strength

$P_p$: Amount of reinforcement for web opening calculated by diameter of prestress tendons arranged near web opening $P_t$: Prestress tensile force per one piece of prestress tendon $P_y$: Yield tensile force of prestress tendon ($=\sigma_y/A$) $\sigma_d$: average prestress suffer to crack occurred near web opening which are assumed that crack angle against longitudinal direction is equal to 45 degree;

$$\sigma_d = n \cdot P_t \cdot \cos(\theta_p) / (b \cdot (\sqrt{2} \cdot D - H) / A)$$

$n$: Number of prestress tendons which cross section having 45 degree of inclined angle $\theta_p$: Inclined angle of prestress tendons against longitudinal axis of specimens

Fig. 1(a) Details of specimens (part 1)

Fig. 1(b) Details of specimens (part 2)
force, which may be varied by reinforcing method, the authors defined \( \sigma_D \) as a average prestress suffer to crack occurred near web opening which were assumed that crack angle against longitudinal direction was equal to 45 degree shown in Fig. 2.

On the IC reinforcing method specimens, specimens had 0.23%, 0.38% and 0.54% of ratio of reinforcement for web opening \( p_p \) calculated by diameter of prestress tendons arranged near web opening.

Fig. 4 shows loading setup. Specimens were suffered from multi-cyclic shear-bending force. Peak point of each loading cycle were decided by deformed angle of specimen \( \mathbf{R} \); \( \mathbf{R} \) were decided to \( \pm 1/500 \) rad., \( \pm 1/200 \) rad., \( \pm 1/100 \) rad., \( \pm 1/67 \) rad. and \( \pm 1/50 \) rad..

To observe crack near web opening, the authors decided the area of 330mm×330mm near web opening, shown in Fig. 4, and width of crack occurred in that area were measured frequently. Crack width \( W \) was defined as width perpendicular to longitudinal direction of crack which occurred diagonally.

3. Result and discussion
3.1 General behavior

Experimental results are shown in Table 3, and relations between shear force \( Q \) and deformation angle \( \mathbf{R} \) and crack pattern are shown in Fig. 5. On specimens of the IC reinforcing method width of crack occurred near web opening became wide and specimens became failure.

On spec. No.1 and No.2, unbonded-type tendons yielded and reached to maximum shear force just before \( \mathbf{R} \) was equal to \( +1/67 \) rad. But on other specimens of the IC reinforcing method, shear force became maximum at the point that \( \mathbf{R} \) was equal to \( +1/67 \) rad. On spec. No.3, specimen of the OC reinforcing method, shear failure were occurred on upper and lower area of web opening when \( \mathbf{R} \) was equal to \( -1/50 \) rad., then \( Q \) had fallen.

Relations between shear crack strength near web opening \( \tau_{sc0} \) and average prestress \( \sigma_D \) are shown in Fig. 6. As figure shows, generally \( \tau_{sc0} \) increased according to growth of \( \sigma_D \), that is growth of prestressed tensile

![Table 2. Mechanical properties of steel bars and concrete](image)

<table>
<thead>
<tr>
<th>Steel bar</th>
<th>( \sigma_{ys} ) N/mm²</th>
<th>( \rho_{Fs} ) N/mm²</th>
<th>( \sigma_{yt} ) N/mm²</th>
<th>( E_s ) GPa</th>
<th>( A ) mm²</th>
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<td>Longitudinal bars D22</td>
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<td>Stirrup U8</td>
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<td>943</td>
<td>195</td>
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<td>PS Tendon for the IC method 7.1Φ</td>
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<td>PS Tendon for the IC method 9.2Φ</td>
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<td>PS Tendon for the OC method 9.2Φ</td>
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<td>195</td>
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\( \sigma_{ys} \): Yield strength  
\( \sigma_{yt} \): Tensile strength  
\( \rho_{Fs} \): Compressive strength

\( E_s \): Elastic modulus  
\( A \): Section area

![Fig. 2. Definition of \( \sigma_D \)](image)

![Fig. 3. Relations between \( \sigma_D \) and \( p_p \) in each spec.](image)

\(^*\): by Takasaki et al. (2002)

![Fig. 4. Loading setup](image)

(Hatched area shows crack-observing area)
Table 3. Experimental result

<table>
<thead>
<tr>
<th>Spec. No.</th>
<th>cQbu kN</th>
<th>cQAL kN</th>
<th>cQAS kN</th>
<th>cQsuo kN</th>
<th>eQmax kN</th>
<th>eQmax/cQsuo</th>
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<td>No.12</td>
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<td>507</td>
<td>1.29</td>
<td>507</td>
<td>1.29</td>
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</tr>
</tbody>
</table>

- $cQbu$: Calculated maximum bending strength
- $cQAL$: Allowable shear strength for sustained loading by AIJ approved formula for reinforced concrete beam not having web opening
- $cQAS$: Allowable shear strength for temporary loading by AIJ approved formula for reinforced concrete beam not having web opening
- $cQsuo$: Shear strength calculated by the Hirosawa’s formula:

$$cQsuo = \frac{0.092 \cdot k_u \cdot k_p \cdot (F_c \cdot + 18)}{M / Q \cdot d + 0.12} \cdot \left(1 + \frac{1.61 \cdot H}{D}ight) + 0.85 \cdot \sqrt{p_p \cdot \sigma_y + p_p \cdot \sigma_y} \cdot b \cdot j$$  \hspace{1cm} (2)

- $p_p$: Ratio of reinforcement for web opening by prestress tendons arranged near web opening:

$$p_p = \sum (a_i \cdot (\sin \theta_p + \cos \theta_p)) / (b \cdot c)$$ \hspace{1cm} (3)

- $a_i$: Sectional area of prestress tendons
- $\theta_p$: Inclined angle of prestress tendons against longitudinal axis of specimens
- $\sigma_y$: Yield strength of prestress tendon
- $eQ_{suo}$: Shear cracking load near web opening taken in experiments
- $eQmax$: Maximum shear strength taken in experiments

![Fig. 5. Q-R relations and crack patterns](image-url)
force. And figure shows that $\tau_{sco}$ may not be influenced by $p_p$ and concrete compressive strength $\sigma_B$.

After shear crack occurred, number of shear crack near web opening became smaller according to growth of $\sigma_D$, and other shear cracks were occurred outside of web opening area.

### 3.2 Crack width near web opening

Relations between average shear stress $\tau$ and maximum crack width $W_{max}$ are shown in Fig. 7, 8, 9 and Fig. 10. On those figures range of $\tau$ were positive. As figure shows, crack width, both width when shear loading and unloading, became small according to growth of $\sigma_D$ and $p_p$. For example, if $\sigma_D$ is equal to 1.7MPa, that is to say that $\sigma_D$ is a small value, crack width became about 1/20 as small as that if $\sigma_D$ is equal to zero, when specimen is under situation of allowable shear stress for long sustained loading. And as figure shows, difference of concrete compressive strength may not be influenced to relations between prestressed tensile force and $W_{max}$. As compared between spec. No.2 and No.3, strength when shear crack occurred were approximately equal, but crack width of spec. No.3 was smaller than that of spec. No.2. Reason why that may be that stiffness for axial direction of prestress tendon used for spec. No.3 was larger than that of spec. No.2, because tendon for spec. No.3 had larger area of section and shorter length than those of spec. No.2.

Relations between $\sigma_D$ and crack width when specimens were under situation of allowable shear stress for long sustained loading are shown in Fig. 11.

Generally in Japan, maximum crack width when prestressed concrete structures are under situation of allowable shear stress for long sustained loading may be recommended to 0.2mm or below. But as shown in figure, crack width of specimens which had no prestressed tensile force for prestress tendon reached between 1.10mm and 1.31mm. Those values are approximately six times as large as 0.2mm. So those facts show that it is estimated that maximum crack width of normal reinforced concrete beam with web opening, not having the IC or OC reinforcing method, may be exceed to 0.2mm, when beam is under situation of allowable shear stress for long sustained loading.

### 3.3 Behavior of reinforcing bar near web opening

Relations between shear force $Q$ and vertical shear force loaded by prestress tendons $V_p$, vertical shear force loaded by stirrup arranged near web opening $V_s$ are shown in between Fig. 13 to Fig. 18. As figure shows, point that shear force loaded by stirrup arranged near web opening became to grow may appear later according to growth of prestressed tensile force. That is because large prestressed tensile force prevents to become width of shear crack near web opening wide.

Among specimens not having prestressed tensile force, characteristics between $\sigma_D$ and $V_p$, $V_s$ are similar to each other even if concrete compressive strength may vary. But on specimen having $\sigma_B$ is equal to 66MPa and having prestressed tensile force, shear force not only loaded by prestress tendon but also loaded by stirrup arranged near web opening tend to grow. So it became clear that prestressed tensile force makes prestress tendon and stirrup arranged near web opening act effectively.

### 3.4 Shear strength

In Japan the formula suggested by Dr. Hirosawa, which is shown in Eq.(1), is well-known for calculation of shear strength of reinforced concrete beam with web opening. Exactly that formula is not following to beam with the OC and IC reinforcing method, but in this section the authors tried to apply that formula to each specimen. Relation between shear strength calculated by the Hirosawa’s formula $Q_{max}$ and that taken in experiments $Q_{max}$ are shown in Fig. 19. In this figure, it is assumed that prestress tendons were considered as reinforcement for web opening like
As figure shows, $e_{Q_{\text{max}}}$ exceeded $c_{Q_{\text{suo}}}$. Value of $e_{Q_{\text{max}}}$, that is safety factor, became between 1.14 and 1.38 on specimens with the IC reinforcing method, and became 1.54 on specimen with the OC reinforcing method.

Safety factor for specimen with the OC reinforcing method was larger than that for specimens with the IC reinforcing method. It may be thought that the reason above is that prestress tendon for the OC reinforcing method has larger effects for shear strength than that of the IC reinforcing method.

4. Conclusion
This study examined the effects of new reinforcing method of reinforced concrete beam with web opening. The following conclusion can be made:
1. Shear crack strength near web opening increases according to growth of prestressed tensile force for prestress tendons arranged or placed near web opening.
2. Crack width become small according to growth of prestressed tensile force for prestress tendons arranged or placed near web opening. Moderate prestressed tensile force for prestress tendons and
amount of reinforcement for web opening may lead crack width under controlled width value.

3. If specimens are made of high strength concrete, prestressed tensile force may make prestress tendon and stirrup arranged near web opening act effectively.

4. The Hirosawa’s formula, which is famous in Japan to estimate shear strength of reinforced concrete beam with web opening, is applicable to specimens having the IC and OC reinforcing methods.

Fig. 12. Shear force at PS tendons and stirrup

Fig. 11. $\sigma_{D Alb}$ vs $W_{max}$ relation

Fig. 13. $V_p$ vs $R$ relation (spec. selected by the scope of variation of $p_p$ and $\sigma_D$)

Fig. 14. $V_p$ vs $R$ relation (spec. selected by the scope of variation of $\sigma_B$ and $\sigma_D$)

Fig. 15. $V_p$ vs $R$ relation (spec. selected by the scope of variation of reinforcing method)
Acknowledgement

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References

1) Takasaki, Y., Katori, K. and Hayashi, S. (2002), Study on Shear Reinforcement for RC Beam with Web Opening under Consideration of Crack Control, Proceedings of the Japan Concrete Institute, Vol. 24, No.2, pp.295-300
2) Architectural Institute of Japan (1987), Data for Ultimate Strength Design of Reinforced Concrete Structures, pp.40-42
3) Akagi, D., Yanase, T., Katori, K. and Hayashi, S. (2003), Experimental Study on Shear Behavior of Prestressed Reinforced Concrete Beams with Web Openings. Proceedings of the Japan Concrete Institute, Vol. 25, No.2, pp.409-414
4) Architectural Institute of Japan (1999), AJI Standard for Structural Calculation of Reinforced Concrete structure
5) Architectural Institute of Japan (1986), Recommendations for Design and Construction of Partially Prestressed Concrete (Class 3 of Prestressed Concrete) Structures
6) Yanase, K., Ohno, Y., Li, Z. and Minami, H. (2002), Shear Crack Width of Reinforced Concrete Beams, Proceedings of the Japan Concrete Institute, Vol. 24, No.2, pp.343-348