Simplified Analytical Model for Outrigger-Braced Structures Considering Transverse Shear Deformation

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Abstract

Several types of simplified analytical model for outrigger-braced structures are proposed. The proposed analytical model is based on the shear-deformable beams with rotational springs. The model can account for the shear deformation of the outriggers as well as flexural deflection of the outriggers and axial deformation of the outer columns. The other simplified model is based on two-dimensional plane frames incorporating shear deformation. Numerical results are obtained for outrigger structural systems addressing the effects of transverse shear deformation. The results by the present models are compared with those of previously available analytical model and full computer model indicating that the present models are efficient and accurate for predicting lateral drift and core resisting moment.

Keywords: Outrigger-Braced Structures; Shear Deformation; Spring Constant; Lateral Drift; Tall buildings

1. Introduction

An outrigger-braced structural system is a common form among tall buildings. It has reinforced concrete or braced steel frame main core connected to the exterior columns directly by flexurally stiff horizontal cantilevers called outriggers.

When an outrigger-braced structure is loaded laterally, the column-restrained outriggers resist the rotation of the core, causing the lateral deflections and moments in the core. This results in increasing the effective depth of the structure when it deflects as a cantilever by introducing tension in the windward columns and compression in the leeward columns (Fig.1).

![Fig. 1. Outrigger-Braced System subjected to Lateral Load: (a) Outrigger System; (b) Lateral Drift; (c) Core Moment](image-url)

Tall buildings are usually so complicated that even an elaborate computational model is a considerable simplification, and the results from an analysis will always be approximate, being at best only as good as the quality of the chosen model and method of analysis. Furthermore, a three-dimensional analysis is necessary if full advantage is to be taken of the special interaction among different elements of the whole structure. Although such an analysis has come within reach as a normal structural design process due to today’s computational technology, its use as an optimization tool may not be desirable in view of time and effort required. In this reason, several simplified analysis techniques have been developed for the analysis of outrigger-braced structures in the past decades.

Smith and his collaborators [1,2] proposed an analytical model for uniform outrigger structures, that is, structures with a uniform core, uniform columns, and similar sized outriggers at each level. They employed a compatibility method, in which the rotations of the core at the outrigger levels are matched with rotations of the corresponding outriggers, to calculate the restraining moments in outrigger-braced structures. Once after the restraining moments are calculated, the whole structure is modeled as a cantilever beam with concentrated moments.

Taranath [3,4] proposed a similar analytical model based on continuum approach. In his model, the core and the outrigger deformations are dependent on the flexural energy changes, while the columns can only store direct force energy. The effect of the outrigger and the columns can be looked upon as being similar...
to that of a moment-resisting spring whose stiffness depends on its location. The rotational stiffnesses of the springs are determined from the axial deformation of the columns and the flexural deformation of outriggers. Accordingly, the structure can be modeled as a cantilever with rotational springs. Jung et al. [5,6] extended Smith model [1] to the outrigger-braced structures under linearly varying lateral loading.

Most studies up to now have concentrated on analysing the outrigger-braced structure neglecting the shear deformation of the core and the outriggers. However, it is well known that the shear rigidity of the braced frame is relatively low, that is, the effects of shear deformation of the core and the outrigger, which is usually a braced frame, become no longer negligible, and sometimes significant. In this sense, the present research proposes two types of simplified analytical models, which incorporate the shear deformation of the outriggers as well as the core. The first model is based on one-dimensional shear-deformable beam with rotational springs, which extend the analytical model by Tanarath [3] by including shear deformation. The other model is based on two-dimensional frame which allows shear deformation.

2. Method of Simplified Analysis

Basic Assumptions
The approximate analysis methods proposed in this study is based on the following assumptions:

- The behavior of outrigger-braced structure is linear elastic.
- The sectional properties of the core, column, outriggers and brace are uniform throughout their height.
- The outriggers are rigidly attached to the core, and pinned connection with the perimeter columns. The core is rigidly attached to the foundation.
- Frames are pinned; thus, the braced core acting in conjunction with the perimeter columns provides total resistance to the lateral load.
- The shear deformation as well as the flexural deformation is taken into account for the core and the outrigger of the structures.

One-Dimensional Beam Model
A one-dimensional analytical model of outrigger-braced structural systems is developed based on shear-deformable beam theory. An outrigger-braced structure is modeled as a shear-deformable cantilever beam with rotational springs as shown in Fig.2.

The rotational stiffness of the spring can be evaluated as reciprocal of the rotation of the outriggers given by

\[ K = \frac{1}{\theta} \]  

(1)

The rotation \( \theta \) consists of three components: that is, rotation due to axial deformation of the columns \( \theta_a \), flexural deformation of the outrigger \( \theta_b \), and shear deformation of the outrigger \( \theta_s \) (Fig.3).
The rotations, $\theta_a$ and $\theta_b$, are expressed, respectively, as follows [1]

$$\theta_a = \frac{2L}{d^2 EA_c} \tag{2a}$$

$$\theta_b = \frac{d}{12 EI_o} \tag{2b}$$

where $EI_o$ is effective flexural rigidity of the outrigger; $EA_c$ is effective axial rigidity of the external column; $d$ and $L$ are total horizontal distance of the outrigger and height of the outrigger from the bottom of the core, respectively.

The rotation due to shear deformation of the outrigger can be calculated by

$$\theta_s = \frac{1}{GA_o h^{'}} \tag{2c}$$

where $GA_o$ is effective shear rigidity of the outrigger, and $h^{'}$ is the depth of the outrigger. While $EI_o$ and $EA_c$ are easily calculated from the building structures, care must be taken to the calculation of effective shear rigidity of the outrigger, $GA_o$. This effective shear rigidity depends on the bracing type of the outriggers as given in Smith and Coull [7]. For example, if the outrigger is single-diagonal bracing, $GA_o$ is given by

$$GA_o = \frac{Eb}{D^2 + a} \tag{3}$$

in which $b$ is story height, $D$ and $a$ are the length of the bracing member and girder, respectively; $E$ is the modulus of elasticity; $A_d$ and $A_g$ are sectional area of the diagonal and the upper girder, respectively; $n$ is the number of the braced frame. For a structure with more than one outrigger, the rotation $\theta$ should be calculated, respectively. Once after the stiffnesses of the rotational spring are determined, the outrigger-braced structures can be analyzed as a shear-deformable cantilever beam with rotational springs.

**Two-Dimensional Frame Model**

The one-dimensional beam model in the previous section is simple, yet it has some shortcomings that the each spring stiffness should be calculated for the outrigger one by one. In this reason, two-dimensional approximate model is also proposed. The model is based on the plane frame element which simplifies the whole structures into a couple of members: only the outriggers and external columns are included in the model as shown in Fig.5.

![Fig. 5. 2-D Approximate Frame Model](image)

Each member can be modeled as a classical beam element or a shear-deformable beam element, and the effective bending, axial and shear rigidities derived in the previous section can be used in the analysis.

**3. Numerical Results and Discussion**

In order to verify the accuracy and efficiency of the present analytical model, an outrigger-brace structure is considered (Fig.6). The results by present approach are compared with the previous result [3] and the finite element package MIDAS [8]. In MIDAS, all the beams and columns as well as the core and the outriggers are modeled in detail, and thus, MIDAS model can be considered to be accurate.

The example structure is a steel-frame structure of 160m total height (40-story) with two outrigger-braces.
Two outriggers are assumed to be located in 80m and 156m from the ground level, respectively. The core and the outriggers are assumed to be of single-diagonal bracing frame, and the outriggers are symmetrically located with respect to the central core.

The elastic modulus is $E = 2.1 \times 10^7$ tf/m$^2$, and the sectional properties of the member are given by:

**External Columns:** H-700x700x32x32  
**Beams:** H-500x200x10x16  
**Bracing and outrigger:** H-500x200x10x16

The effective flexural and shear rigidities are calculated and given in Table 1.

**Table 1.** Flexural and shear rigidities of the structure

| Shear rigidity of the core $(GA)_c$ | 2.2382 x $10^5$ tf  |
| Flexural rigidity of the core $(EI)_c$ | 9.8868 x $10^7$ tf m$^2$  |
| Shear rigidity of the outrigger $(GA)_o$ | 6.7145 x $10^5$ tf  |
| Flexural rigidity of the outrigger $(EI)_o$ | 1.4742 x $10^7$ tf m$^2$  |

Two types of loadings are considered:
Uniformly-distributed load of $q = 1$ tf/m, and linearly varying load of $q_1 = 1$ tf/m and $q_2 = 2$ tf/m. Total degrees of freedom of the analysis models are compared in Table 2 for three different analytical models. It is seen that the present analytical models substantially reduce the degrees of freedom of the structure.

**Table 2.** Comparison of total degrees of freedom for various analytical models

<table>
<thead>
<tr>
<th>Analytical Model</th>
<th>MIDAS</th>
<th>Frame</th>
<th>Beam</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of Nodes</td>
<td>267</td>
<td>13</td>
<td>4</td>
</tr>
<tr>
<td>Number of Elements</td>
<td>476</td>
<td>14</td>
<td>3</td>
</tr>
<tr>
<td>Total D.O.F.'s</td>
<td>1068</td>
<td>52</td>
<td>16</td>
</tr>
</tbody>
</table>

The lateral deflection of the structure under uniformly-distributed load is presented in Table 3 for various analysis models. In computing the one-dimensional beam model result for “core shear considered”, the shear deformation of the core is considered, but the shear deformations of the outriggers are neglected. For “outrigger shear considered”, the shear deformations of the outriggers are considered, but the shear deformation of the core is neglected. For “all shear considered”, both the shear deformations of the core and the outriggers are included in the analysis.

As can be seen in Table 3, the result by Taranath model [3], which neglects all the shear deformation, significantly underestimates the lateral deflection up to 53% compared with the result by MIDAS. In viewing the results of the proposed one-dimensional models, the shear deformation of the outriggers affects more than that of the core on the lateral deflection of the structure. The result with “all shear considered” shows good agreement with MIDAS solution. It is also noted that the result of the two-dimensional frame model with shear considered agrees well with MIDAS solution, while the two-dimensional solution without shear consideration shows discrepancy with MIDAS solution. Fig. 7 shows the nondimensional lateral deflection of the structure for the present analytical models with shear deformation included and previous model and MIDAS. The resulting moments in the core are compared for various analysis models in Table 4. By including the shear deformation of the core, the resulting moments increase.

The lateral deflection of the structure under linearly-varying load is presented in Table 5 and Fig. 8 for various analysis models. Again, Both the one-dimensional analytical model, which considers both the shear deformation of the core and the outrigger, and the two-dimensional simplified frame model with shear effect show excellent agreement with MIDAS solution.
That is, these models can give sufficiently accurate results with considerably less efforts and time compared to three-dimensional full MIDAS model.

![Graph](image)

**Fig. 7.** Lateral deflection of outrigger-braced structure under uniformly-distributed wind load

**Table 4.** Resulting moments in the core for the outrigger-braced structure under uniformly-distributed load [tf·m]

<table>
<thead>
<tr>
<th>Model</th>
<th>Core Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIDAS</td>
<td>7199</td>
</tr>
<tr>
<td>Taranath [3]</td>
<td>6839</td>
</tr>
<tr>
<td>Beam Model</td>
<td></td>
</tr>
<tr>
<td>Core Shear considered</td>
<td>6839</td>
</tr>
<tr>
<td>Outrigger Shear Considered</td>
<td>7501</td>
</tr>
<tr>
<td>All Shear Considered</td>
<td>7501</td>
</tr>
<tr>
<td>Frame Model</td>
<td></td>
</tr>
<tr>
<td>Shear Neglected</td>
<td>7024</td>
</tr>
<tr>
<td>Shear Considered</td>
<td>7662</td>
</tr>
</tbody>
</table>

![Graph](image)

**Fig. 8.** Lateral deflection of outrigger-braced structure under linearly-varying distributed wind load

**Table 6.** Resulting moments in the core for the outrigger-braced structure under linearly-varying load [tf·m]

<table>
<thead>
<tr>
<th>Model</th>
<th>Core Moment</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIDAS</td>
<td>11623</td>
</tr>
<tr>
<td>Taranath [3]</td>
<td>10984</td>
</tr>
<tr>
<td>Beam Model</td>
<td></td>
</tr>
<tr>
<td>Core Shear considered</td>
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</tr>
<tr>
<td>Outrigger Shear Considered</td>
<td>12126</td>
</tr>
<tr>
<td>All Shear Considered</td>
<td>12126</td>
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<tr>
<td>Frame Model</td>
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<tr>
<td>Shear Neglected</td>
<td>11310</td>
</tr>
<tr>
<td>Shear Considered</td>
<td>12409</td>
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</tbody>
</table>
4. Concluding Remarks

Two types of analytical models were developed to study the deflection of outrigger-braced structural system. The model is capable of predicting accurate deflection for various loading conditions. The proposed analytical model is based on the shear-deformable beams with rotational springs. The model can account for the shear deformation of the outriggers as well as flexural deflection of the outriggers and axial deformation of the outer columns. The other simplified model is based on two-dimensional plane frames incorporating shear deformation. Based on the theoretical developments and numerical results, the following concluding remarks can be made:

1. The previous conventional model, which neglects the shear deformation of the core and the outriggers, overestimates the shear rigidity of the core and the rotational spring constants. Accordingly, the model significantly underestimates the lateral deflection of the outrigger-braced structures.

2. The one-dimensional analytical model, which considers the shear deformation of the core, gives better results than the conventional model, yet yields remarkable discrepancy in lateral deflection with MIDAS model due to excluding of the shear effect of the outriggers.

3. The effect of shear deformation of the outrigger is found to be more important to that of the core on the behavior of outrigger-braced structure. That is, the shear deformation of the outrigger should be considered for the accurate analysis of outrigger-braced structure.

4. Both the one-dimensional analytical model, which considers both the shear deformation of the core and the outrigger, and the two-dimensional simplified frame model with shear effect show excellent agreement with MIDAS solution for all the problems considered. That is, the model can give sufficiently accurate results with considerably less efforts and time compared to three-dimensional full model.

The approximate method of analysis is valuable in providing an fundamental understanding of the behavior of a tall building structure and in allowing the initial sizing of primary members as part of the preliminary design process. The proposed analytical models are found to be very accurate and efficient for the analysis of the behavior of an outrigger-braced structure. As a natural extension of this research, a model which incorporates the twisting deformation, varying stiffness along the height, or P-Δ effect awaits further attention.

Acknowledgement

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Reference