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Stability of Diagrid Structures

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Abstract

In this paper, the stability requirements for diagrid and mega braced structures are examined. The role of the secondary bracing system for the stability of a diagrid structure is discussed. A simple procedure is proposed for the design of the secondary bracing system when it is required. As a case study, the design of the Hearst Tower diagrid and its secondary bracing system are presented.

Keywords: Diagrid, Mega bracing, Super diagonals, Stability, Secondary bracing

1. Introduction

Efficiencies in the strength and stability of truss systems have been understood since the Middle Ages. The major impetus for widespread use of the truss system however, came in the 19^{th} century with its implementation in bridge construction, early on in wood and eventually in cast and wrought irons. This created a surge of inventive truss configurations such as the Allen Lattice Truss, Warren Truss and Pratt Truss among many others. The truss structures reached their most impressive height with the construction of the Eiffel Tower in 1889.

The first diagrid structure was introduced by the brilliant Russian engineer Vladimir Shukhov in the design of the world's first hyperboloid diagrid structure in Nizhny Novgorod, Russia in 1896. However, it took sixty years after the introduction of the diagrid structure for the first diagrid building, known as the IBM building in Pittsburg to be constructed in 1963, (Curtis, 1963).

Another forty years passed before diagrid structures became the focus of a new concept in architecture and structural engineering, and for their aesthetic and intrinsic structural value to be appreciated. The diagrid resurgence was led by Foster and Partners Architects and their Structural Engineers in the design and construction of 30 St Mary Axe (Swiss Re) in London, Hearst Tower in New York, and The Bow in Calgary, (Boake, 2014; Rahimian and Eilon, 2007). Since then many other diagrid buildings have been proposed, designed and constructed, (Boake, 2014).

Similarly, the use of truss and mega bracing followed its own trajectory after the construction of the Eiffel tower. The John Hancock Center in Chicago, engineered by Fazlur Khan and constructed in 1969, was the first application of mega bracing in tall buildings while the Bank of China Tower (367 m) in Hong Kong reached new heights with its impressive application of mega bracing in 1990, (Blake, 1991).

Diagrid and braced structures are both variants of truss structures where the primary mode of load transfer is by axial stress in which materials show their maximum efficiency and economy in resisting forces. A structure with mega bracing, whether in the form of a diagrid or individual bracing, requires special design considerations as relates to its stability. This is due to the fact that mega bracing elements spanning over multiple floors may not provide lateral support for the structure at each of the floors that it passes through. The bracing elements provide lateral stiffness as part of a truss system at their triangulated “hard panel” nodes. At any other point along the brace member the lateral stiffness would be a function of the flexural stiffness of the bracing element acting as a beam and not as a truss system. Therefore, depending on the size and geometry of the diagonal elements they may need to be laterally stabilized at other levels in addition to the hard panel node levels. A procedure for the stability design of the bracing is explained in this article. A Diagrid system does not meet the limitation of the Direct Analysis Method of AISC as it requires that “*The structure supports gravity loads primarily through nominally-vertical columns, walls or frames*.” However, the AISC Direct Analysis method is examined here and it is demonstrated that the method is not applicable for a diagrid system with multistory elements.

2. Classifications

There are fundamentally two solutions for the stability design of diagrid and mega bracing elements. One solution is to consider the full length of the bracing element

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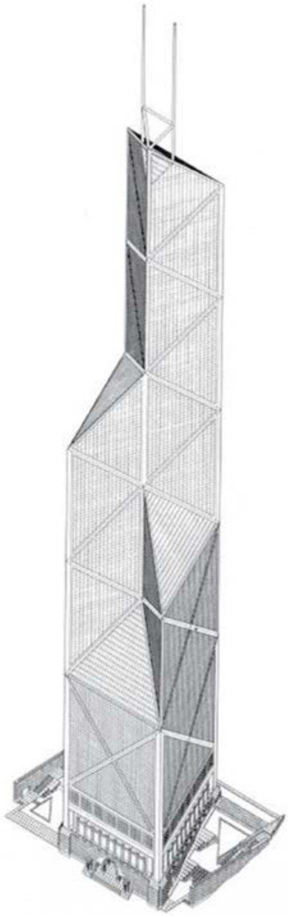

Figure 1. Bank of China courtesy of Pei Cobb Freed & partners.

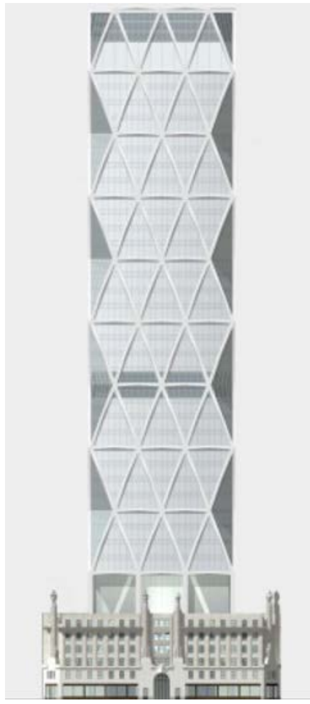

Figure 2. Hearst Tower Courtesy of Foster and Partners.

between the hard panel nodes spanning over multi floors as the un-braced length. This approach generally requires large cross sectional dimensions relative to the span of the element in order to provide adequate buckling capacity as well as sufficient lateral stiffness to control the inter-story building sway for the floors between the nodal floors. The Bank of China and Swiss Rae tower are two extreme example of this approach where the bracing elements and/ or the mega columns flexural stiffness provide adequate rigidity to allow those elements to span between the hard panel nodes, see Fig. 1. In this approach no secondary bracing system is required.

The second approach considers the bracing elements to be laterally supported at the levels away from the hard panel nodes by a secondary bracing system (SBS). The secondary bracing system itself will also be considered laterally supported by the mega brace system at the hard nodal levels. Hearst Tower is an example of this approach, see Fig. 2.

This paper addresses the secondary bracing requirement for the diagrid or mega brace system. The diagrid system designed for Hearst Tower is used as an example.

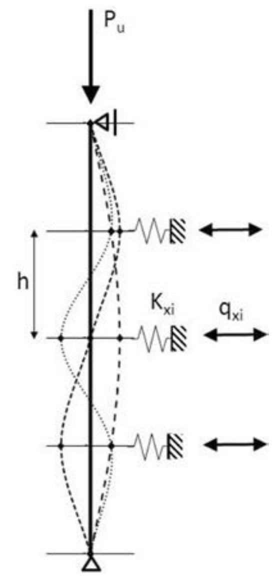

Figure 3. Buckling modes of a multistory column with lateral spring supports.

3. Secondary Bracing System

3.1. Column Lateral Bracing

The lateral bracing of columns with intermediate lateral supports as shown in Fig. 3 have been studied by Winters and others based on equilibrium requirements of a column

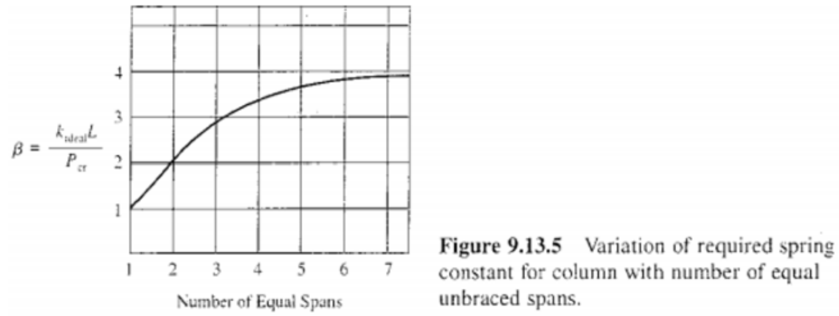

Figure 4. Required spring constant for stability of a multistory column, from Salmon and Johnson, 1980.

under a laterally displaced position considering the so called P-D effect, (Abdelrazaq et al., 1993; Galambos, 1998; Geschwindner and Lepage, 2013; Montuori et al., 2014; Salmon and Johnson, 1980; Winter, 1960; Yura, 1996).

The following equations show the stiffness and strength requirements at the laterally braced nodes.

$$k_x = \beta \cdot P_u / h(1+\Delta_0/\Delta) \tag{1}$$

Where $1 \leq \beta \leq 4$ is a parameter to account for the buckling mode shape as shown in Fig. 4, (Salmon and Johnson, 1980). The SBS stiffness K_x should attain the largest possible value per Eq. (1).

The Δ_0 is the initial column out of plumbness at the bracing nodes. The term $(1+\Delta_0/\Delta)$ represents the amplification effect of the construction imperfection. Winter suggested considering $\Delta_0=\Delta$ as reflected below.

$$k_x = 2 \cdot \beta \cdot P_u / h \tag{2}$$

Eq. (3) shows the required strength of the SBS system.

$$q_u = k_x \cdot \Delta = 2 \cdot \beta \cdot P_u \cdot \Delta / h \tag{3}$$

The AISC Code of practice recommends considering a construction tolerance of h/500 between the splice points, (AISC 303, 2010). Eq. (4) shows the required strength of the SBS system considering Δ_0=h/500 as the construction imperfection and considering that the amplified deformation is set at $\Delta_0=\Delta$.

$$q_u = k_x \cdot \Delta = 0.004 \cdot \beta \cdot P_u \tag{4}$$

3.2. Stability of the Diagrid

By applying the same principal of equilibrium as discussed above to the diagrid structure in Fig. 5 the following stiffness requirements for SBS for the stability of the diagrid can be obtained. In formulating equilibrium it should be considered that the diagrid axial force, while it is in a vertical plane, is also inclined by an angle of α as shown in Fig. 5. The required stiffness is a function of the magnitude of the diagrid compression force. Eq. (5) shows the required stiffness of SBS for each diagrid under the load applied at the hard point of the diagrid as shown by P_u.

$$k_x = 2 \cdot \beta \cdot F_u / (h \cdot \sin(\alpha)) \tag{5}$$

Where, F_u represents the axial force in the diagrid and α is diagrid angle from the horizontal axis.

Considering a complete triangular diagrid as shown in Fig. 5 the required SBS stiffness at each level can be writ-

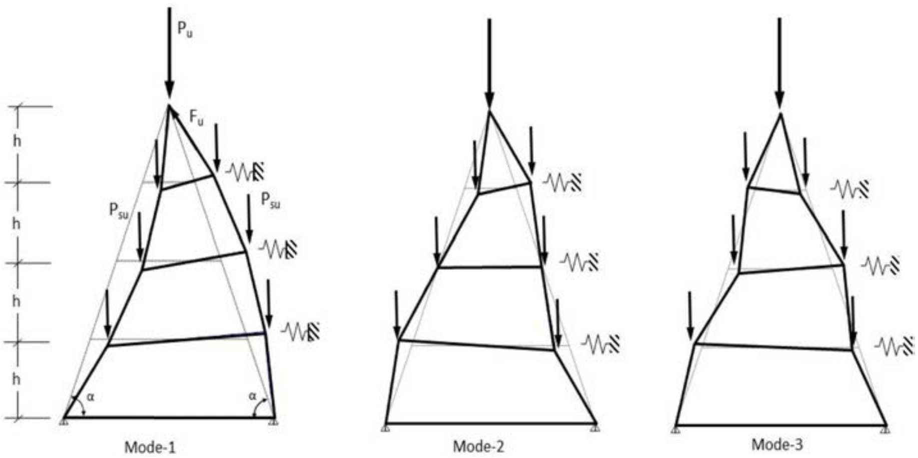

Figure 5. Buckling deformation shape of a single diagrid module.

ten as;

$$k_x = 2 \cdot \beta \cdot P_u / (h \cdot \sin^2(\alpha)) \tag{6}$$

Eqs. (7) and (8) show the required lateral stiffness and strength for a series of interconnected diagrid systems respectively.

$$k_x = 2 \cdot \beta \cdot \Sigma P_u / (h \cdot \sin^2(\alpha)) \tag{7}$$

$$q_u = k_x \cdot \Delta_x = 0.004 \cdot \beta \cdot \Sigma P_u / \sin^2(\alpha) \tag{8}$$

Where, ΣP_u is the sum of the vertical component of all diagrid axial forces measured at a specific level.

Eq. (9) combines the stiffness requirements of Eqs. (2), and (7) to account for the stability of the entire system at a floor level located away from the diagrid hard panel nodes.

$$k_x = 2 \cdot \beta \cdot \Sigma P_u / (h \cdot \sin^2(\alpha)) + 2 \cdot \beta \cdot \Sigma P_{lu} / h \tag{9}$$

Where, ΣP_{lu} is the total vertical load of the leaning columns at a specific level. The first term in Eq. (9) is for the stability of the diagrid element in the plane of the diagrid, the second term is for the stability of the leaning columns and the out of plane diagrids.

Likewise, the strength requirement for the SBS system can be obtained similar to Eq. (10) as shown below by combining the requirements of Eqs. (4), and (8).

$$q_u = 0.004 \cdot \beta \cdot \Sigma P_u / \sin^2(\alpha) + 0.004 \cdot \beta \cdot \Sigma P_{lu} \tag{10}$$

4. Design of the Secondary Bracing System

For the design of the SBS system as described above, the strength requirement from Eq. (10) can be converted to a set of lateral forces to be applied to the SBS system. For this purpose, the deformation shapes representing the possible initial imperfection and buckling modes need to be considered. The buckling mode shapes can be obtained from an eigenvalue analysis of the diagrid structure with representation for leaning elements and the secondary bracing system. The eigenvalue analysis for buckling requires incorporation of the geometric stiffness matrix. Alternatively, for a system with identical "Nodal" bracing (springs) or identical "Relative" bracing stiffness at all levels the mode shapes can be assumed to follow a sine wave, similar to an Euler buckling shape of a prismatic column (Winter, 1960; Geschwindner and Lepage, 2013).

Subsequently, a geometric nonlinear (P-Delta) analysis of the system can be performed for each of the buckling modes. The profile of the mode shapes scaled to an AISC construction tolerance of h/500 is considered as an initial imperfection in order to obtain the lateral forces for the stability of the system. A four story diagrid which has three intermediate "flexible" levels would have three modes of buckling.

The geometric nonlinear analysis can be performed either using structural analysis software such as SAP or ETABS or via a relatively simple computation based on Eqs. (9) and (10) without incorporating the β factor, as the actual β factor will be calculated internally through the stability equilibrium analysis for each of the mode shapes.

The results of the analysis have been confirmed with eigenvalue buckling analysis using SAP2000 and ETABS 15. The analyses also verified that the buckling mode shapes for nodes at the floor levels can be idealized as a sinusoidal curve when the SBS spring constants and the floor heights are equal at all levels.

For computational simplicity the stiffness demand for the SBS system in the preceding equations is represented as series of independent nodal springs. In the actual design of structures the SBS system would be envisioned as a moment frame, vertical truss or a wall system spanning vertically between the hard panel node levels of the diagrid system. As a result, the actual stiffness effect of the SBS system is referred to as "relative bracing", since at each level the lateral stiffness of the SBS will have interdependency with the adjacent levels. In other words, the stiffness matrix representing the SBS lateral stiffness will have off-diagonal non-zero elements. Fig. 6(a) shows the stiffness matrix for the lateral degrees of freedom of the structure with nodal bracing for a structure as shown in Fig. 3 while Fig. 6(b) shows the stiffness matrix for the lateral degrees of freedom of the structure with relative bracing for a structure as shown in Fig. 7. k_{ij} represents the relative bracing stiffness at each level and its value is ½ of k_{ii} for a uniform system. The fundamental difference between the nodal and relative bracing is the existence of the off-diagonal stiffness elements k_{ij}. Further explanation can be found in Geschwindner and Lepage, 2013.

Fig. 7 represents a vertical braced frame as the secondary bracing system. For the cases where the inter-story stiffness of the SBS system is identical at all levels the stiffness matrix and geometric stiffness matrix will become proportional, (Zhang et al., 1993). As a result, for a given relative (inter-story) stiffness all three lateral modes of buckling will converge to one eigenvalue and therefore to one buckling load, P_u. In this scenario the ideal relative (inter-story) stiffness of the SBS system will be $k=k_{ij}=$ Pu/h. The stiffness k is the off-diagonal stiffness matrix value and Pu is the total vertical load at a given level. As discussed before, assuming that the structure lateral displacement is equal to the initial imperfection ($\Delta=\Delta_0$) the

$$\begin{bmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & k_3 \end{bmatrix} \quad ; \quad \begin{bmatrix} k_{11} & k_{12} & 0 \\ k_{21} & k_{22} & k_{23} \\ 0 & k_{32} & k_{33} \end{bmatrix}$$

(a) (b)

Figure 6. (a) Stiffness Matrix for Nodal Bracing, (b) Stiffness Matrix for Relative Bracing.

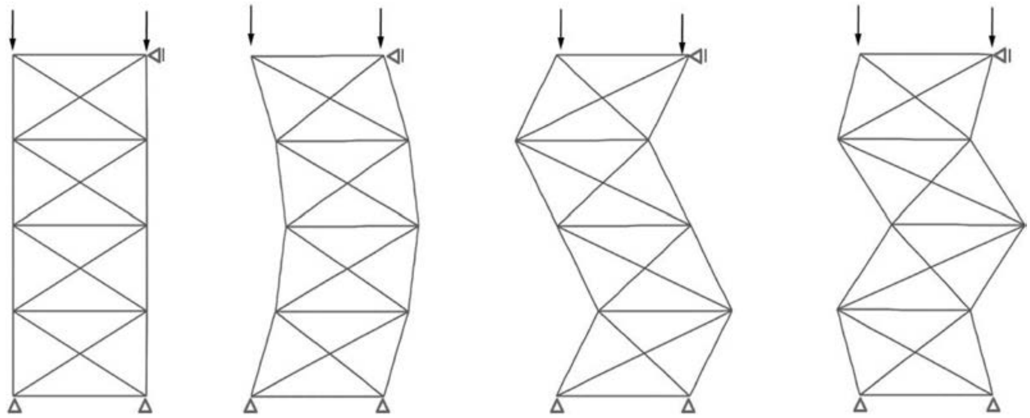

Figure 7. Buckling deformation shape of a secondary truss system.

required inter-story stiffness will be $K=2P_u/h$.

The ideal floor stiffness of the SBS at each level, which is also the diagonal stiffness matrix value, will be $K=k_{ii}=2k_{ij}=2P_u/h$. Similarly, the required floor stiffness ($\Delta=\Delta_0$) will be $K=4P_u/h$.

All three modes of buckling are possibilities that should be considered as part of the design of the secondary bracing elements and their connections regardless of the type of bracing; nodal or relative.

5. Case Study: Hearst Tower

As an example, the stability analysis of a four story module of the diagrid structure of the Hearst Tower between the 18th and 22nd floors is presented. Fig. 8 shows a four story module representing the diagrids, interior columns and secondary bracing arrangement used in the Hearst Tower. Fig. 9 shows a 2D model of a diagrid at of the East facade with representation for leaning columns and a leaning orthogonal diagrid as well as the first line of secondary bracing truss representing the eastern half of the floor.

This study considers that the four story module supports an additional 24 stories above. A total vertical load of 14,000k on the diagrids and 17,000k of vertical load on the leaning columns and 10,000k of vertical load on the leaning diagrids are considered to represent the load from the floors above imposed on the eastern half of structure at the 22nd floor. For simplicity the local floor loads at levels 18 through 21st are ignored in this study. The P-Δ equilibrium analysis as explained above is performed for each mode of buckling for the loads explained above.

The required secondary bracing and strength for each of the modes of buckling and the mode shape profiles are shown in Table 1. The analysis is performed for both nodal bracing and relative bracing methods. The mode shapes in the table are numbered based on the period of the sinusoidal curves, the single curvature being the lowest mode. This also represents the lowest spring constant values required for the buckling resistance. However, if performing eigenvalue for a set of given spring constants the order of buckling is reversed as the 3rd mode would provide the lowest buckling load capacity. To obtain the required strength q_u as shown in Table 1 the mode shapes are scaled to an AISC maximum inter-story tolerance of h/500.

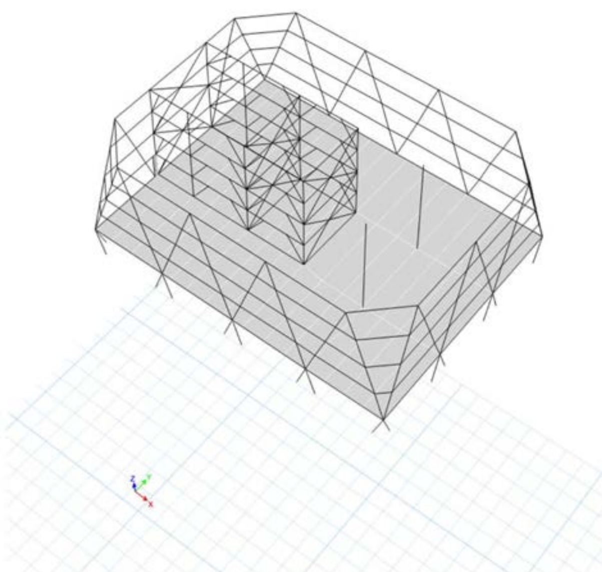

Figure 8. Isometric view of a four story segment of the diagrid structure of Hearst Tower.

The last column in Table 1 called "Relative SBS" shows the required lateral floor stiffness considering relative bracing stiffness. This represents the k_{ii} (diagonal) values in the stiffness matrix shown in Fig. 6(b). The relative stiffness value k_{ij} is ½ of k_{ii} for a uniform system. The q_u strength requirements for each of the mode shapes shown in Table 1 are also applicable for the relative bracing system as well as the nodal bracing system.

Subsequently, the secondary truss system is designed to satisfy the strength and stiffness requirements shown in

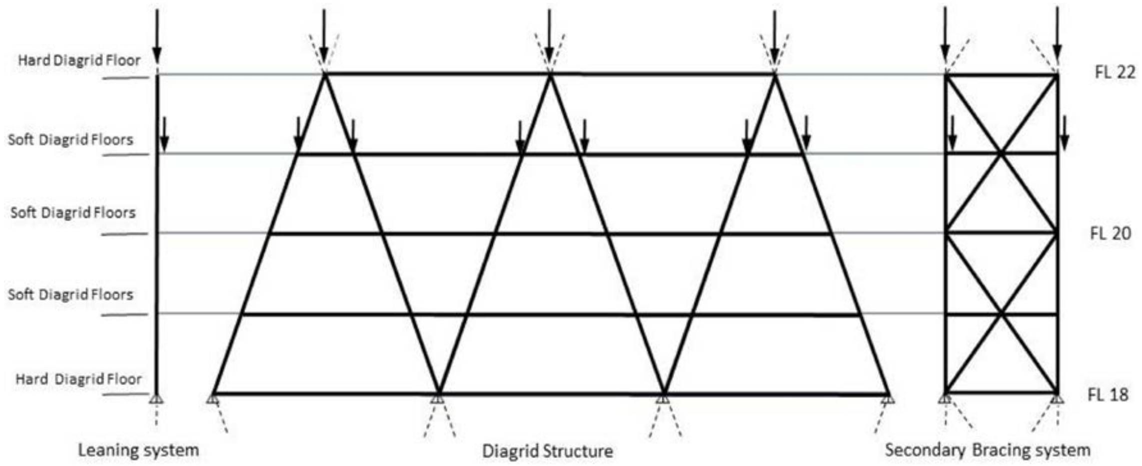

Figure 9. A 2D diagram representing a four story segment of the diagrid structure, leaning structure and secondary bracing system.

Table 1. The diagonals for the SBS system are designed for each of the three sets of stability forces shown on Table 1 in combination with gravity and other loads. These can be applied as an additional set of loads for the design of the SBS bracing elements.

While only a four story segment is studied here as an example, the stability analysis for the project was expanded to take into account the entire building structure.

Depending on the relative position of the SBS truss within the building, the effect of the eccentricity of the SBS system with respect to the center of gravity and stiffness of the floor needs to be accounted for by performing a 3D stability analysis. In addition, the floor diaphragm needs to be designed to ensure adequate strength and stiffness for load transfer between the SBS, the diagrid structure, and the leaning systems.

Fig. 10 shows the lateral displacement of the structure under the P-Δ effect and Fig. 11 shows the inter-story deformation obtained from the stability analysis as compared to the criteria of h/500.

6. AISC Provisions

The direct analysis method recognizes that the stability and out-of plumbness requirement can be, in principle, addressed by adding a notional load for the design of the elements, (AISC 360, 2010).

However, the AISC limits the application of the direct analysis method to structures with nominally vertical columns. Therefore, it precludes a diagrid structure. The following study explains the fundamental shortcomings of the Direct Analysis Method and the notional load approach for a diagrid structure.

The notional load prescribed by AISC is modeled after the first mode of buckling of a free standing cantilever structure for a homogeneous structural system such as a moment frame or truss system. A homogenous system is defined here as structural system that has a similar load path at each floor in the transfer of lateral forces to the floor below. The notional load, which is proportional to the vertical load imposed at each level, will produce a story shear that will pass through all elements of the lateral system and impact all elements in a similar manner. However, a multistory diagrid structure or a mega brace system that is not connected at each level to the panel nodes defies this principle since the notional load story shear at the nodal panel level will be converted to a series of diagrid axial forces with no flexural or shear effect at

Table 1. Buckling mode shapes, stiffness and strength requirement for nodal and relative bracing

Mode	1	2	3	Nodal spring 1	Nodal spring 2	Nodal spring 3	1	2	3	Relative SBS
Level	Mode shape	Mode shape	Mode shape	krqd (k/in)	krqd (k/in)	krqd (k/in)	Qu (k)	Qu (k)	Qu (k)	krqd (k/in)
22	0.00	0.00	0.00				-172	-172	-71	
21	1.00	1.00	0.41	310	1060	1809	101	343	243	1060
20	1.41	0.00	-0.59	310	1060	1809	142	0	-343	1060
19	1.00	-1.00	0.41	310	1060	1809	101	-343	243	1060
18	0.00	0.00	0.00				-172	-172	-71	

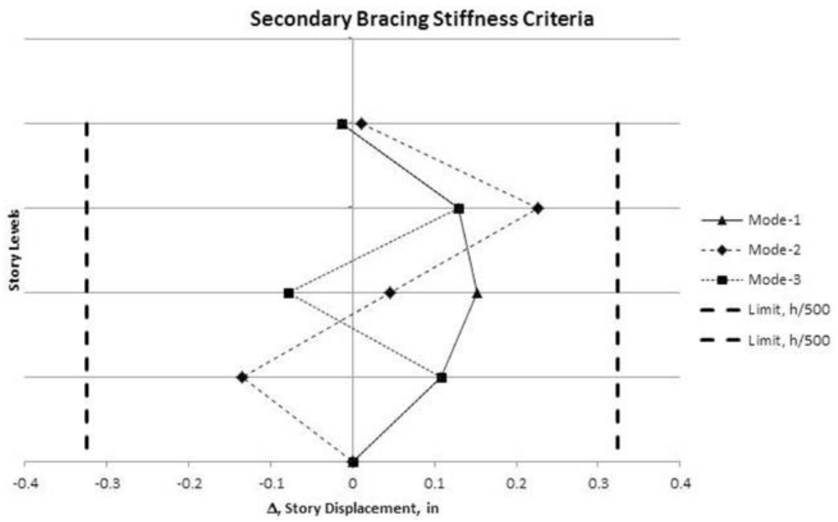

Figure 10. Buckling modes of the diagrid structure.

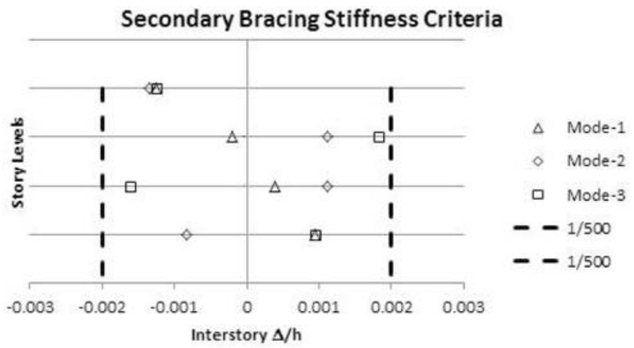

Figure 11. Lateral deflection of the diagrid structure under initial imperfection and P-Δ effect.

Table 2. Notional load at hard panel node level

Level	Applied Floor Load, k	Vertical Force at each level, k	Notional Load, k
22	41000	41000	82
21	0	41000	0
20	0	41000	0
19	0	41000	0
18	0	41000	0

levels between the panel nodes as illustrated in Fig. 12. As a result neither the diagrid system nor the secondary bracing system, SBS will be exposed to the cumulative story shear effect of the notional load.

As an example, Table 2 shows the notional load for the four story diagrid module as shown in Fig. 9. As can be seen from Fig. 12 the notional load from floors above, when it reaches the hard panel nodes of the diagrid, is converted to a series of axial forces in the diagrid system. As explained above, for simplicity of discussion the local floor loads are ignored. As a result the notional load for the local floor is also ignored as the magnitude of the load is significantly smaller than the load from the floors above. Per AISC provisions the sum of notional load applied at the 22nd floor would be yi= 0.002*41,000= 82k. It should be noted that the notional loads for floors 19, 20 and 21 even if they had been considered would have been about yi=.002×1700=3.4 kips for a local floor load of 1700 kips. In addition, as explained above the notional load story shear at the nodal panel level will be converted to series of axial forces in diagrids with no flexural or shear effect at levels between the panel nodes (levels 19, 20 and 21).

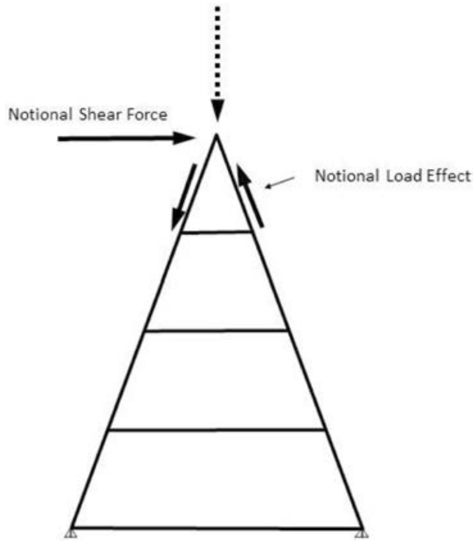

Figure 12. AISC notional load at the hard panel node level.

Therefore a comparison of the notional loads with the lateral force obtained from the stability analysis indicates that the notional load method does not adequately address the stability requirement of a multistory diagrid system. In addition, as explained above, the AISC limits the application of the direct analysis method to structures with fairly vertical columns and as a result AISC does not recommend that the Direct Analysis Method be considered for diagrid structures.

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