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Author: Sam Lee, Guangzhou Scientific Computing Consultants Co. Ltd.

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Nonlinear Dynamic Earthquake Analysis of Skyscrapers

Sam Lee

Guangzhou Scientific Computing Consultants Co. Ltd, 507/140 Dongfeng Xi Rd, Guangzhou 510170, China, Email: szslee@gmail.com

szslee@gmail.com

Biography

Dr. Sam Lee studied building structural design at Tsinghua University in China, receiving his double bachelor degrees of engineering mechanics and structural engineering in 1988. He spent eight years working for The Architectural Design and Research Institute of Guangdong Province, Guangzhou China. During this period, he was involved in the structural design and documentation of more than 30 tall buildings, most of which exceeded 100m in height, including the Qingdao China Construction Bank (33 stories, 100m tall) and the Guangan Masion (150m tall) in Hainan Province.

In 1996, he moved to Sydney, Australia with his family and joined Henry & Hymas as a structural engineer. He developed a tilt-up panel lifting program call TiltMAX, where firstly, the optimum lifting point locations are identified and finally using FEA method to analyze the panels and obtain accurate stress distribution in the panel during lifting. The program is currently in use in many Australian structural consulting firms.

In 2004, he was awarded a PhD degree from the school of civil and environmental engineering, University of New South Wales, Sydney. His major research areas included: (1) Semi-analytical structural analysis methods base on partial differential equations ;(2) Fast algorithms for nonlinear dynamic structural analysis ;(3) Nonlinear programming for structural analysis ;(4) The nonlinear dynamic analysis of tall buildings. He also developed a program called BEPTA (Building Elastic-Plastic Time history Analysis) on ABAQUS platform, specific for the nonlinear dynamic analysis of skyscrapers.

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Guangzhou Scientific Computing Consultants Co. Ltd, 507/140 Dongfeng Xi Rd, Guangzhou 510170, China, Email: szslee@gmail.com

Abstract

Due to the limitations of computer capacity and the softening of the material constitution, until now the nonlinear dynamic earthquake analyses of skyscrapers has not been practical in engineering practices, and even in the research area there had still been open problems. In this paper, a solution is provided on the ABAQUS platform by selecting the right analysis procedure, accurate nonlinear models for the structural members, an efficient dynamic equation integration scheme and the appropriate earthquake records. In the analysis model, all members and reinforced concrete shear-walls are modeled by plastic zone model, and large deflection effects are taken into account. Especially, as the shear-walls are divided into element sizes of around 0.7x0.7m, the material and geometry nonlinear buckling behaviors of the shear-walls are modeled almost numerically exact. The main analysis procedures and some key parameters are outlined. The analysis results of some prominent projects in China, such as Shanghai World Financial Center (492m in height), Jinta (330m in height) and Guangzhou West Tower (435m in height), are presented. The analysis results are of great help for the design engineers to study the skyscrapers earthquake performance and therefore design the skyscrapers' structure with added safety and economy.

Keywords: Nonlinear, Dynamic, Skyscraper, Earthquake, ABAQUS

1. Introduction

Seismic design is a very critical issue for skyscrapers built in a seismic area. In the earthquake engineering area, the earthquake loads are usually classified into three levels by their intensities (GB50011, 2001): frequently level; design intensity level; severity level. For the first earthquake load level, the linear analysis is adequate. However, the skyscrapers go into plastic range when subjected to the second and third earthquake intensity levels, therefore a more vigorous analysis (nonlinear dynamic analysis) should be applied.

Nonlinear dynamic earthquake analysis is the most adequate and comprehensive analysis procedure to evaluate the nonlinear seismic response of structures, but currently available computer hardware and design software effectively limit the size and complexity of structures that may be analyzed using this technique. At present, there is no general-purpose nonlinear analysis software that will permit practical evaluation of large structures that include elements with the wide range of inelastic constitutive relations actually present in the building inventory (FEMA274, 2001). As a matter of fact, most (if not all) current design engineering practices use the linear procedures only to do the structural design, and moreover, use the seismic concept design technique such as seismic fortification measures and details of seismic design to make the skyscrapers perform well under the design intensity and rare level earthquake load. It means that the design engineers don't know how the skyscrapers will response to the design intensity and rare level earthquakes. However, even in the research area, it is still an open problem.

There are three barriers preventing the engineer from the application of nonlinear dynamic earthquake analysis. The first being, it was just too complex to be solved using earlier computers. The second is the results might be very sensitive to the properties of the structural nonlinearities. The last is the analysis results vary significantly to the different earthquake records. In this paper, a solution is provided on the ABAQUS platform by selecting the right analysis procedure, accurate nonlinear models for the structural members, an efficient dynamic equation integration scheme and the appropriate earthquake records. Nonlinear dynamic earthquake analysis becomes a practical analysis tool for structural design, and because of the technique, the skyscraper can be designed with added safety and economy.

In the following section, the procedure of nonlinear dynamic earthquake analysis is introduced. Then the nonlinear model for the structural members is presented, in particular, the nonlinear constitution for steel and concrete are briefed, as the nonlinearities of the structural members are based on the material stress-strain relationship level. In the next section, the dynamic equation integration scheme is outlined. Emphasis is placed on how the explicit scheme can solve the large and complex problem. How to select the right earthquake records and the damping issues are discussed in the next sections. In the following section, a software package call BEPTA (Building elastic-plastic time history analysis), which has been developed on the ABAQUS platform which specializes in all kinds of tall buildings, is introduced. The nonlinear dynamic earthquake analysis procedure is carried out by running BEPTA and

ABAQUS in tandem. The results judgment and evaluation method is presented in the section 7.

A couple of project examples are presented in last section. These show that even the largest and most complex projects in China can be analyzed in detail. Finally, some conclusions are drawn for nonlinear dynamic earthquake analysis.

2. Analysis procedure

The nonlinear dynamic analyses of skyscrapers consist of two major steps that run in a tandem: (1) gravity load analysis according to the construction sequence; (2) seismic load analysis based on the stress statues obtained in the first step. Each step solves the highly nonlinear problems.

The skyscrapers are built up one story at a time. The gravity load analysis step should model this procedure using couple of substeps, each substep represents a construction step (for example, one story) as shown in Fig.1.

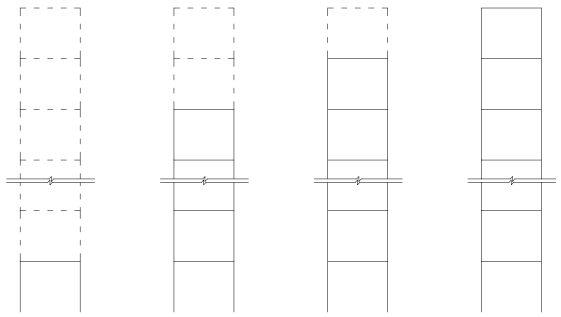


Fig.1 Construction sequence

The whole model is built up in the computer first. In the first step, all structural elements above the first story are "killed" (shown as dot line), the stress status is obtained. The next steps are to progressively activate the structural elements above one story. Using this process, all structural elements are activated and the final stress status is obtained at the completion of the construction sequence. While the structural materials (steel and concrete) might remain in the elastic range under the gravity load, the analysis step requires that the stiffness matrix of the model be updated for each substep. In fact, the stress status when analyzed using the construction sequence method is significantly different than that of a one off loading analysis, in particular the beams' bending moments at the top of the building.

The construction sequence analysis is a quasi-static procedure, and it is suitable to be carried on by a nonlinear static solver. It should be mentioned that if some the member sizes are not designed or input properly, it is difficult to converge and the analysis aborts.

3. Nonlinear model

The nonlinearity of the structure includes geometry nonlinearity, material nonlinearity and a combination of both.

3.1 Geometry nonlinearity

Geometry nonlinearity can be modeled accurately by use of the Green strain formula. The P-Δ effects and large deflection effects are automatically taken into account. Most general finite element analysis packages have this built-in function available.

3.2 Material nonlinearity

Steel and concrete are the basic materials used in the structural elements. To model the cyclic characteristics of the earthquake load, a nonlinear material model with specific cyclic features should be used for each.

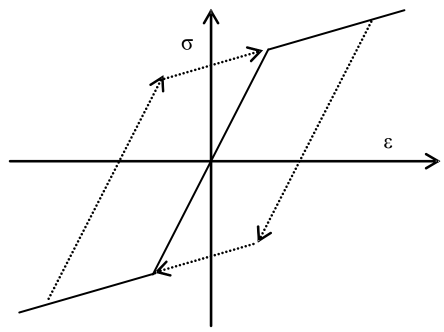


Fig.2 Steel constitute law

3.2.1 Steel

In this article, an isotropic kinematic hardening model is used for steel material. As shown in Fig.2, the Bushinger effect has been taken into account, and there is no stiffness degradation during the cycling. It is acceptable for the skyscraper structure as the maximum steel strain should be less than 2.5%.

3.2.2 Concrete

The plastic-damage model (J. Lee, 1998) is used to model the concrete material. The model is a continuum, plasticity-based, damage model for concrete. It assumes that the main two failure mechanisms are tensile cracking and compressive crushing of the concrete material. It captures the three major characteristics of the concrete in the buildings: (1) the strength of compression is larger than that of tension; (2) the stiffness degrades when it goes into plastic range; (3) the stiffness recovers when it reverses from tension to compression.

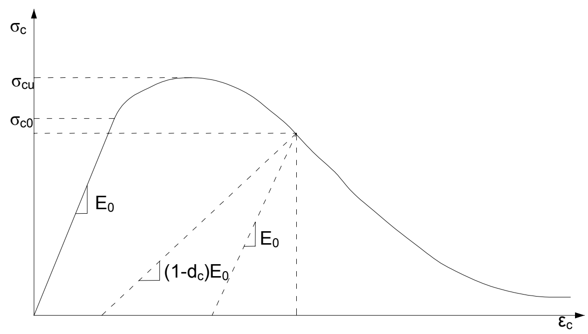


Fig.3 Concrete in tension

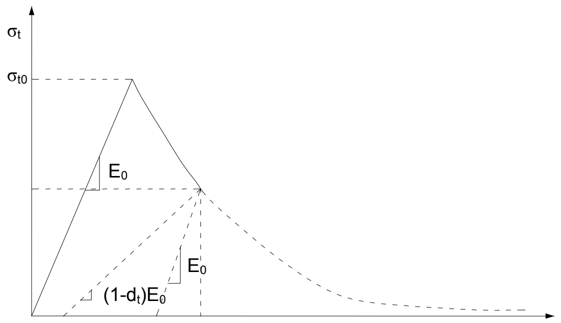


Fig.4 Concrete in compression

Fig.3 and Fig.4 show the concrete material's stress-strain curve, the stiffness of the concrete degrades when it unloads from the plastic range. The degradation factors for compression (d_c) and tension (d_t) are dependent on the plastic strain (ABAQUS, v6.5). Fig.5 shows the hysteric curve of the concrete, it can be seen that the stiffness recovers when the material stress status reverses from tension to compression.

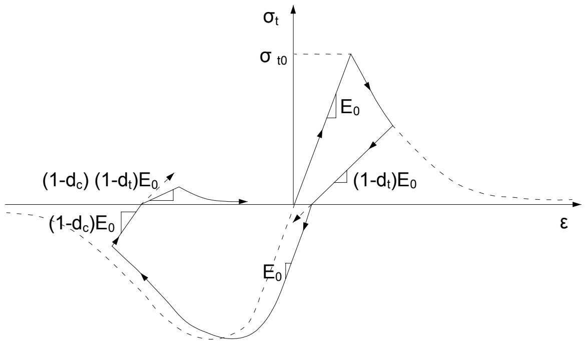


Fig.5 Concrete hysteric curve

3.3 Member model

Most structural elements are line elements and shell elements. Line elements are used to model beams and columns, while shell elements are used to model the steel or reinforced concrete shear walls.

3.3.1 Line elements (beam, brace and column)

For line elements, rigid section plane assumption is used. The section is dissected into multiple fibers, as shown in Fig. 6.

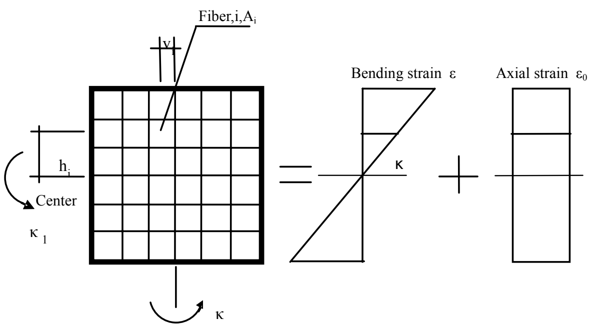


Fig.6 Fiber model for line element

The fiber can be steel or concrete. The strain of fiber "*i*" can be obtained in terms of κ_1, κ_2 and ε_0:

$$\varepsilon_i = \kappa_1 \times h_1 + k_2 \times v_1 + \varepsilon_0$$

Therefore the section bending moments and axial forces are as follows:

$$N = \sum_{i=1}^{n} A_i \times f(\varepsilon_i), M_1 = \sum_{i=1}^{n} A_i \times f(\varepsilon_i) \times h_i$$
$$M_2 = \sum_{i=1} A_i \times f(\varepsilon_i) \times v_i$$

Where $f(\varepsilon_i)$ is obtained by the material constitutions. It should be pointed out that the axial forces of beams are rather large when it goes into plastic range and can not be ignored. Therefore the interaction of bending and axial forces should be considered.

As the plastic zone model is adopted, the stiffness of the line element is dynamically obtained by integrating in sectional and longitudinal directions. The hysteric features of the members are represented by the cyclic features of the materials. As show in the Fig.7, the fibers go in to plastic gradually in sectional and longitudinal directions.

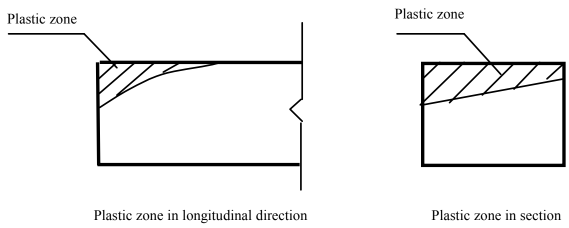


Fig.7 Fiber plastic zone model

A 3D Timonshenko beam element is used. The more divisions of a physical beam or column, the more accurate the results obtained. To compromise the accuracy and the computing capacity, four and more divisions for each physical beam or column is used, as shown in Fig. 11.

It should be noted that the shear stiffness is assumed constant, as the shear failure is brittle and is not allowed in structural design. The back check shear forces against the shear capacity should be carried on after the nonlinear dynamic earthquake analysis is done.

It should also be noted that the plastic zone model shown here is numerically exact for line element, but it takes enormous computer resources.

Due to the limitation in computer resources in the past, some simplified nonlinear models based on the force resultants-deflections were used. Currently all structural design software including SAP2000, ETABS, MIDAS, etc, are using these kinds of models. For examples, plastic hinge model for beams and P-M1-M2 nonlinear model for columns (shown in Fig.8) assume

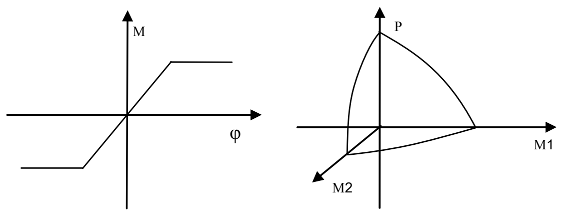


Fig.8 Plastic hinge model and P-M1-M2 model

that the plasticity occurs in the whole section suddenly while other sections remain elastic, and their nonlinearities are represented by section bending moments-rotation angles and axial forces-axial strains, which is section size and shape related. These simplified models are suitable for calculating the limit bearing capacity of the members, but when they are used to analyze cyclic load and the post yielded conditions, the errors are significant compared to the plastic zone model.

In fact, more than a hundred models have been proposed for different force resultants-deflection relationships by the researchers, those models might work only at their specific load and section range. There are so many models (there can be millions in theory) and each model can lead to different analysis results, the engineers can become easily confused. This might be an obstacle for the application of nonlinear dynamic earthquake analysis for skyscrapers.

The plastic zone method used in the article, which is based on the material constitution level which is determined by the material only, removes the above difficulties associated with the simplified method. With rapidly increasing computer speeds and the emergence of new efficient algorithms, the plastic zone model is now the best model for nonlinear dynamic earthquake analysis.

3.3.2 Shell elements

A general-purpose, three-dimensional, first-order shell element that uses reduced integration with plastic-damaged concrete material for concrete reinforcement is used to model concrete shear walls and slabs, while the same shell element with steel material is used to model the steel shear wall.

Each node of the shell element has six degrees of freedom that is easy to connect to the line elements. To accurately model the shear wall and slab, the size of shell element is meshed to about 0.7m by 0.7m. The distributed rebar layer can also be taken into account for concrete reinforced shear walls and slabs.

It should be mentioned that most structural design packages still can not provide nonlinear shell elements. The shear walls are simplified to be a line element or frame. However, due to the complexity of shear core walls, the results of simplified models significantly differ from the nonlinear shell element results in elastic range, let alone when the shear wall goes into the plastic range. Therefore results of the simplified model are hard to justify theoretically and should be treated with caution.

To increase the ductility of the reinforced concrete shear wall in the bottom of the building, some reinforced columns or steel braces are built-in the shear wall, as shown in Fig.9. The line elements are used to model the built-in columns and braces, and share the same nodes with the shear wall shell elements.

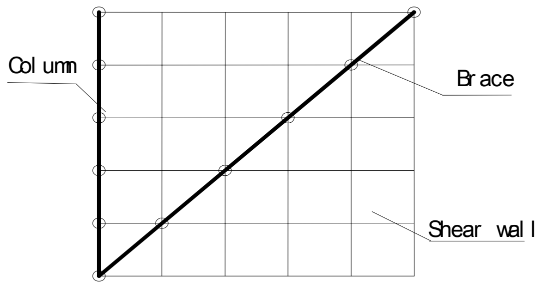


Fig.9 The modeling of column built-in the shear wall

The link beams which connect to the shear walls play very important roles in the dissipation of earthquake energy. It will go into plastic range first and then the whole stiffness of the buildings changes. When its height span ration is large, it might shear yield in plan. In this case the link beam is modeled as a shell element, as illustrated in Fig. 10.

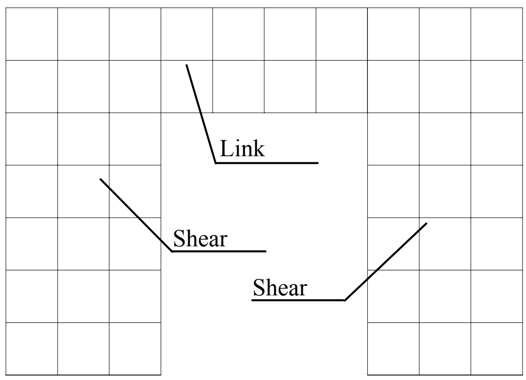


Fig.10 Link beam modeling

Finally, the structural system is built up by the connections of beams, columns, braces, slabs and shear walls. As shown in Fig.11, the structural members are subdivided into 4-6 sections to capture the geometry and material nonlinearities of the structure.

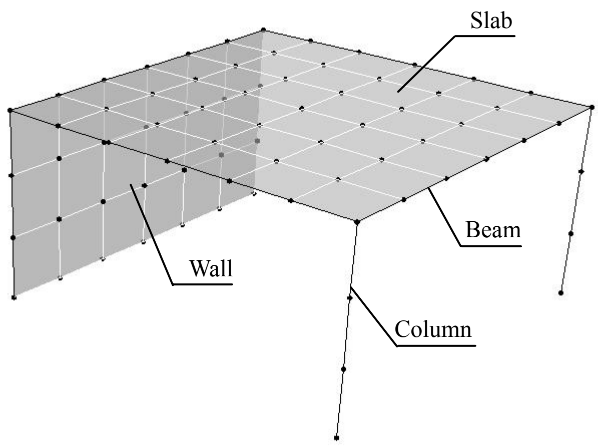


Fig.11 Finite element model for beam, column shear wall and slab

4. Integration of the nonlinear dynamic equations

It's well known that the dynamic equation for the structure is as follow:

$$\mathbf{m}\ddot{\mathbf{x}} + \mathbf{c}\dot{\mathbf{x}} + \mathbf{f} = -\mathbf{m}\ddot{\mathbf{u}}_{\mathbf{g}}$$

Because of the nonlinearity of the structure, the mode-based dynamic analysis method is not suitable and the direct integration dynamic analysis should be applied. Implicit scheme and explicit scheme are the two major integration methods. Implicit scheme is used solving for dynamic quantities at time t based not only on values at t, but also on these same quantities at $t+\Delta t$, while explicit scheme uses a central difference rule to integrate the equations of motion explicitly through time, using the kinematic conditions at time t to calculate the kinematic conditions at $t+\Delta t$.

Generally speaking, the unconditional stable implicit scheme is used to solve structural dynamic problems. Newmark method is one of the most widely used implicit schemes. However, two problems are difficult to solve when the implicit scheme applies to the nonlinear dynamic earthquake analysis: (1) the implicit scheme requires inversion of the stiff matrix in each increment, but the time to reverse the matrix increases exponentially as the size of the matrix increases. Even for a small problem with 5 million degrees of freedom to integrate 20s, its analysis time is unacceptably long; and (2) when some members of the structure are severely nonlinear, it must subdivide the increment to get the problem converged. Moreover, when the stiffness of the structure changes abruptly or negative stiffness occurs, the problem still can not converge even though very small increments are used.

The explicit scheme is conditionally stable and requires an increment stability limit less than the highest frequency of the system and much less than the implicit scheme. However, each increment is relatively inexpensive (compared to implicit scheme) because there is no solution for a set of simultaneous equations and without requiring tangent stiffness matrices to be formed. The calculation time linearly increases with the number degree of freedoms, as shown in Fig. 12. The explicit scheme has super advantage for large scale problems. Moreover, because the small increments are used, the earthquake loading can be modeled more precisely, and the divergence problem can be avoided when the structure goes into the severe plastic range.

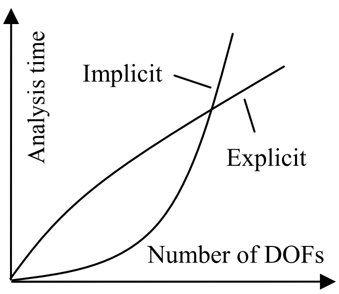


Fig.12 Comparison of explicit and implicit

5. Earthquake records and damping ratio

The earthquake records are provided by the seismic engineer based on the site investigation. The response spectrum of the earthquake records should comply with the code specific design spectrum. The peak ground acceleration shall be determined by the design intensity of the earthquake and the site classification. The duration of the earthquake records must be longer than 4-6 times of the basic period of the building.

It should be noted that most earthquake records available can not meet the code's spectrum requirement at periods longer than 4s, which are the cases for the skyscrapers. In this case, the artificial earthquake records which comply with code specifications should be used.

The earthquake always comes in three directions (one vertical and two horizontals). Therefore three earthquake records are input into the structure in an analysis run.

The material damping ratios are applied in the nonlinear dynamic analysis. According to the code, 5 percent damping ratio is used for concrete building and 2 percent damping ratio is used for steel structural building.

6. BEPTA Program

The scale of the nonlinear dynamic analysis of skyscrapers is very large. It is quite often that the degrees of freedom of the model are larger than a million. To handle the large amount of data correctly and smoothly, a strong and robust pre-post process program is mandatory for the analysis. BEPTA (Building elastic-plastic time history analysis) is a program developed on the ABAQUS platform, specific to the nonlinear dynamic analysis of skyscrapers. It includes the following functions:

1. Suitable to do elastic or elastic-plastic analysis for all kinds of structures such as steel structure, steel-concrete mix structure and concrete structure.
2. Automatically transfer the structural data such as geometry, material elastic properties, member section and reinforcement into ABAQUS input deck.
3. Automatically set the nonlinear analysis control parameters.
4. Automatically set the nonlinear material parameters for steel and concrete.
5. Automatically set the analysis procedure specific to nonlinear dynamic analysis of skyscrapers.
6. Develop a concrete user subroutine for 3D beam element, which is not available in ABAQUS and other general purpose FEA codes.
7. Post process the super large analysis results, and produce analysis report specific to skyscraper nonlinear dynamic analysis.
8. The flowchart of the nonlinear dynamic analysis of skyscrapers, as illustrated in Fig.13, show the relationship of BEPTA with ABAQUS and some popular structural design programs, such as SATWE and ETABS.

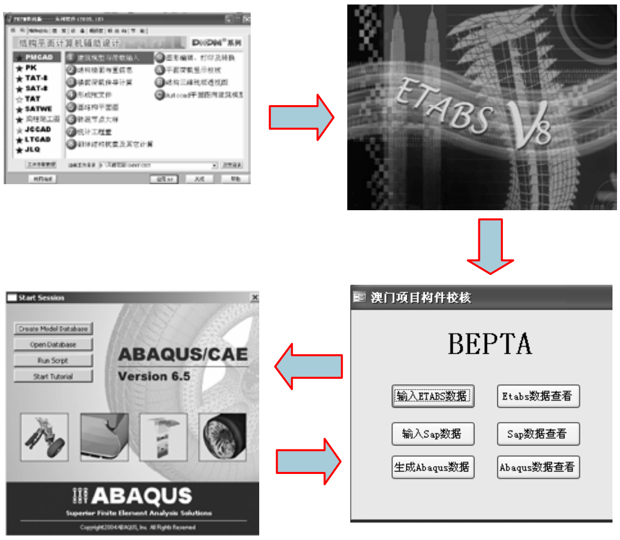


Fig.13 Flow chart of BEPTA

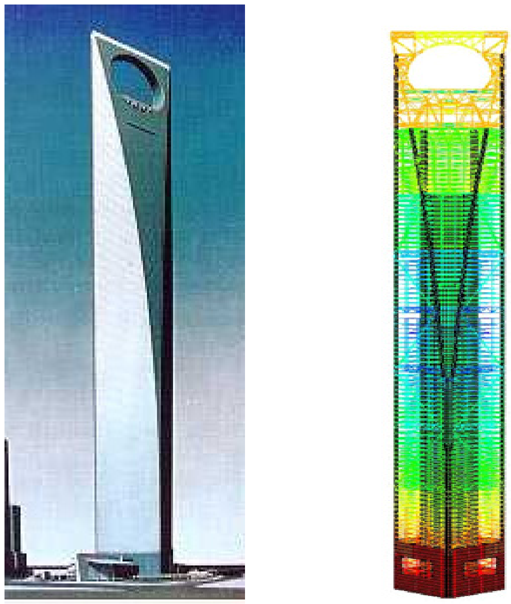


Fig.14. The architect impression and structural system of Shanghai World Financial Center

(Courtesy of KPF Architect and Leslie Robertson Associates)

7. Results judgment and evaluation

As the nonlinear dynamic earthquake analysis is almost the true simulation of skyscrapers under the earthquake action, the judgments of the result is quite straight forward. Three criterions are used to evaluate the performance of the building:

1. The structural integrity on completion of analysis. i.e The structure can still carry the gravity load although some structural elements may have large plastic deformations. This is the basic performance requirement for the structure.
2. The ductile factor (maximum strain/elastic strain) should be less than the value prescribed by FEMA356.
3. Some critical structural members (Say the core wall of the building) should remain in elastic state. The critical structural members are determined by the structural design engineer based on their design goals.

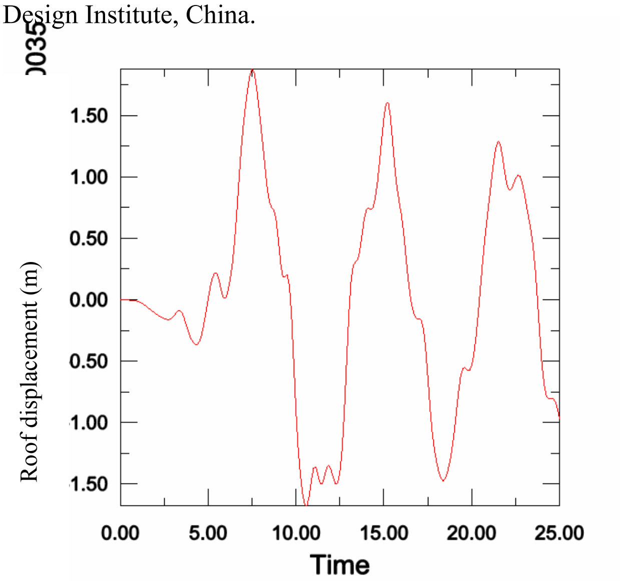
8. Project examples

By making use of BEPTA+ABAQUS programs, more than 20 skyscrapers have been analysed using nonlinear dynamic analysis. The results comply with the rules of mechanics and agree with the engineer's design goals. Among them, four prominent projects in China are shown as follows.

8.1 Shanghai world financial center, Shanghai, PRC.

The architect's perspective view is illustrated as Fig.14. The skyscraper is 492m tall at its roof, and has 101 floors in total. As shown in Fig.14, the concrete core walls and the perimeter mega steel frames form the lateral system of the structure. Without simplification, all the structural members such as beams, columns, braces and shear walls have been modeled by the corresponding elements.

The degrees of freedom of the model are 600K. The output file size is 6G with 200 steps results. The cluster computer (6 CPUs) analysis time is 30 hours. Fig.15 shows the roof displacement time history; the maximum roof drift is 1.6m, 1/307 of the total height. The analysis was undertaken in accordance with the structural design provided by the East China Architecture Design Institute, China.



Time (s)

Fig.15. Roof displacement time history

8.2 Jinta, Tinjin, PRC.

The architect's perspective view is illustrated as Fig.16. The skyscraper is 350m tall at its roof, and has 75 floors in total. As shown in Fig.17, the steel shear walls and the four outriggers form the lateral system of the structure. Without simplification, all the structural members such as beams, columns, braces and steel shear walls have been modeled by the corresponding elements. Fig.17 shows the plastic strain contour of the structure.

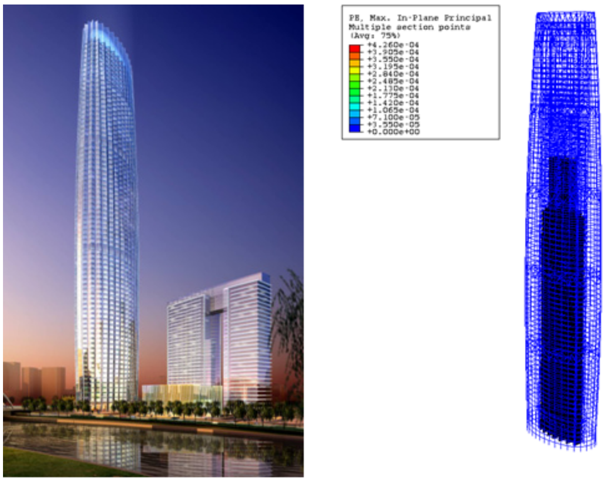


Fig.16 & 17. The architect impression and structural system of Jinta (Courtesy of SOM)

Fig.18 shows one elevation's plastic deformation. The steel shear walls are plastic buckling under the earthquake action. The maximum plastic strain is 2.485e-3, which is much less than the limit prescribed by FEMA356.

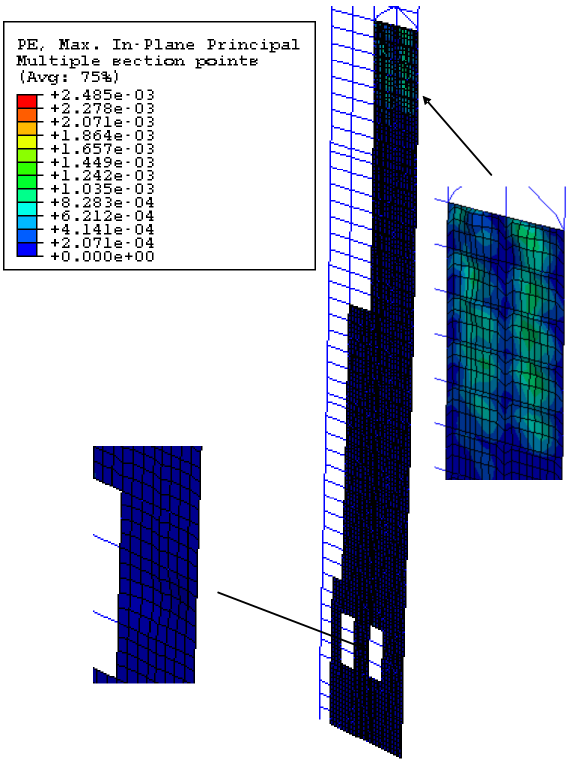


Fig.18. The plastic deformations of one elevation

8.3 West Tower, Guangzhou

The architect's perspective view is illustrated as Fig.19. The skyscraper is 435m tall at its roof, and has 108 floors in total. As shown in Fig.19, the concrete core walls and the slant perimeter concrete filled tube grids form the lateral system of the structure. Again, all the structural members such as beams, columns, braces and shear walls have been modelled by the corresponding elements.

Fig.19. The architect impression of Xita (Courtesy of WilkinsonEyre/Arup)

The structure performs well under the rare earthquake. Fig.20. shows the compression damage of the concrete core at the transfer level. It can be seen that some shell elements are heavily damaged, but in overall the core wall can still carry the vertical load.

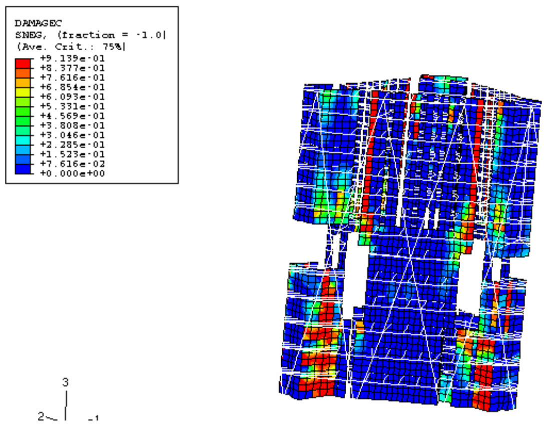


Fig.20. The compression damage of core wall.

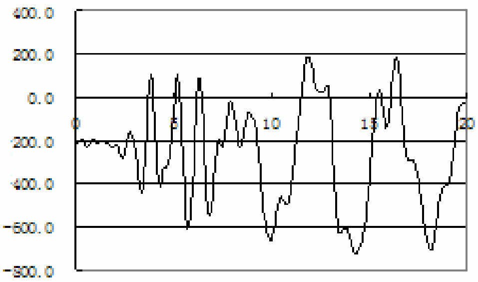
8.4 CCTV new headquarters, Beijing.

The architect's perspective view is illustrated as Fig.21. The two 6 degree slant towers are connected by a 14 story roof structure. The cantilever is 70m at a height of 200m. It is probably the oddest shape building in China. The outer braces and the steel frames in each elevation are the major lateral force resistant system. Again, all the structural members such as beams, columns, braces and shear walls have been modelled by the corresponding elements.

The vertical displacement of the tip of the over hang under a large earthquake is of most concern. Fig.22 shows that a maximum 0.7m downward displacement occurs at 15s.

Fig.21.The architect impressin of CCTV

(Courtesy of OMA/Arup)



Time(s)

Fig.22. The vertical displacement at the hang's tip

Some steel braces in the elevation go into plastic buckling during the earthquake, as shown in Fig.23. The buckling braces dissipate the seismic energy thus allowing the major vertical members to carry the vertical load.

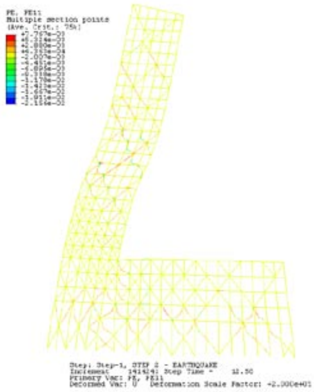


Fig.23. The plastic deformation of south elevation

9. Conclusions

The treatment of the nonlinear dynamic earthquake analysis of skyscrapers is presented in the paper. First the construction sequence analysis must be carried out and then the earthquake analysis runs in tandem. The plastic zone models based on the material stress-strain relationship are used to model the structural members. These models have removed the theory errors that other simplified models have, and can be regarded as numerically exact. Utilizing the explicit scheme to integrate the nonlinear dynamic equations, the large and highly nonlinear dynamic problem is solved practically and efficiently. The appropriate earthquake records which should comply with code specifications and site investigation should be used. The structural damping ratio is presented.

A program call BEPTA (Building elastic-plastic time history analysis), which is developed on the ABAQUS platform specifically for nonlinear dynamic analysis of skyscrapers, is introduced. The judgments of the results are straight forward and no new terms developed. Four prominent projects in China are analyzed by BEPTA and outlined.

The following conclusions can be drawn from the paper's work:

1. Nonlinear dynamic earthquake analysis with numerically exact nonlinear models for skyscrapers is practical. The computer analysis time for a project is about one day, which can be used practically in the preliminary design stage.
2. The explicit scheme to integrate the dynamic equation is a feasible method to solve the large and complicated problems with the complex and numerically exact model used to model the nonlinearity at stress-strain level.
3. As the stress-strain level nonlinear model is used, the results sensitivity to the material nonlinear and the earthquake records is much less significant compared to other simplify models.
4. The BEPTA+ABAQUS solution is a practical way to implement the nonlinear dynamic earthquake analysis.

The extensions of this work will include consideration of the semi-rigid connections between beams and columns in steel structures and a simulation of buildings under terrorist attacks.

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