

Title: **Shear Behavior of R/C Beams with Web Openings Reinforced by Prestress Force**

Authors: Shizou Hayashi, Professor, Tokyo Institute of Technology
Keiichi Katori, Assistant Professor, Tokyo Institute of Technology
Daisuke Akagi, Technical Staff, Takenaka Corporation

Subject: Structural Engineering

Keywords: Concrete
Shear
Structure

Publication Date: 2004

Original Publication: CTBUH 2004 Seoul Conference

Paper Type:

1. Book chapter/Part chapter
2. Journal paper
3. **Conference proceeding**
4. Unpublished conference paper
5. Magazine article
6. Unpublished

Shear Behavior of R/C Beams with Web Openings Reinforced by Prestress Force

Keiichi Katori¹, Shizuo Hayashi² and Daisuke Akagi³

¹ Assistant Professor, Structural Engineering Research Center, Tokyo Institute of Technology

² Professor, Structural Engineering Research Center, Tokyo Institute of Technology

³ Technical Staff of Nagoya Branch, Takenaka Corp.

Abstract

In tall building, there is a trend towards providing openings through the webs of reinforced concrete (R/C) beams to accommodate service ducts. But R/C beams with web openings have sectional loss caused by web opening. So sometimes severe cracks having large width may appear near web opening, and those severe cracks may affect seismic performance of R/C beams.

To improve those situations, the authors have proposed new reinforcing method for web opening of R/C beams. That new method is to introduce prestress force to area around web openings; the force is loaded by prestress tendons arranged near web openings. In this study, the authors performed shear-bending experiments of R/C beam specimens to investigate seismic characteristics of beams having such reinforcing methods.

From experiments the following fact made clear; (1) Strength of specimens when shear crack occurred near web opening grew according to growth of introduced prestress force for tendons, (2) When prestress force was introduced, crack width near web opening could be controlled by introduced prestress force for tendons, and (3) Shear ultimate strength of specimens could be estimated by the formula.

The authors confirmed that the proposed new methods were effective for reinforcing of R/C beam with web opening.

Keywords: Reinforced Concrete, Beam, Web Opening, Prestress, Tendons

1. Introduction

In tall building construction, there is a trend towards providing openings through the webs of reinforced concrete (R/C) beams to accommodate service ducts. R/C beams with web opening have section loss caused by web opening, so their section loss may cause large crack width around web opening. Sometimes their large crack width may exceed beyond design assumption. To improve that situation, the authors has proposed new reinforcing method, named “the IC (inner confinement) reinforcing method; the IC reinforcing method is made up of unbonded-type prestress tendons arranged in inside of beams around web opening and introducing prestress tensile force to prestress tendons. The authors intend that the IC reinforcing method may be adopted for new construction buildings for restraint and control of crack

width being small.

In this research the authors intended to investigate effects of amount of prestress tensile force to prestress tendons, ratio of web opening reinforcement by prestress tendons and compressive strength of concrete, for ability of restraint of crack width and shear ultimate strength of reinforced concrete beams having the IC reinforcing method.

The authors, furthermore, have proposed new reinforcing method, named “the OC (outer confinement) reinforcing method”, for beams with web openings in existing building; the OC reinforcing method is made up of prestress tendons placed on the surface of beam roundly, and introducing prestress tensile force to the tendons. In this study the authors also intended to investigate effect of the OC reinforcing method.

2. Experimental Program

Specimen's details are shown in *Fig. 1*, and *Table 1*. Twelve specimens were tested in this study. Eleven of specimens were made as specimens of the IC reinforcing method, and one of specimen was as the OC reinforcing method. All specimens had 300mm in

Contact Author: Keiichi Katori,
Assistant Professor, Structural Engineering Research Center,
Tokyo Institute of Technology,
4259 Nagatsuta, Midori, Yokohama, Kanagawa 226-8503
Japan
Tel: +81-45-924-5338 Fax: +81-45-924-5526
e-mail: katori@serc.titech.ac.jp

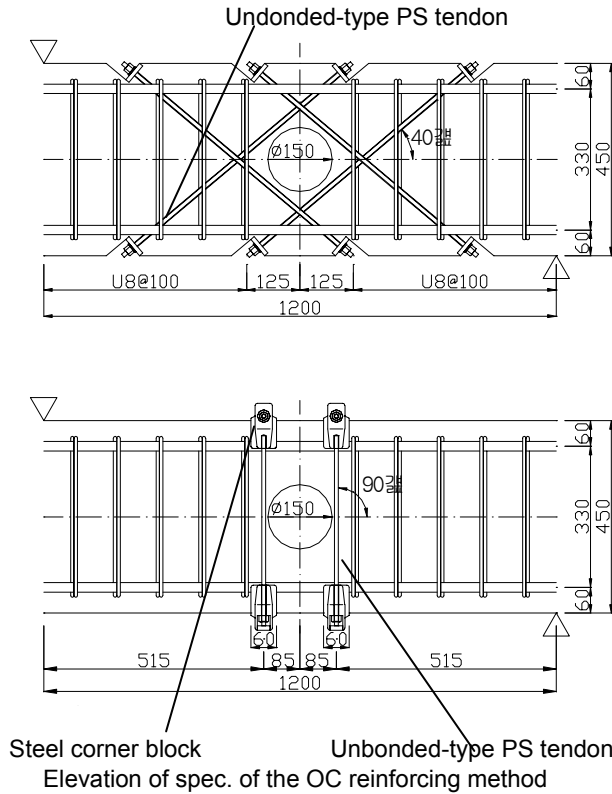


Fig. 1(a) Details of specimens (part 1)

width b and 450mm in depth D of section, 150mm in diameter of web opening (equal to $D/3$) and 1.54 of shear span ratio M/Qd .

Six pieces of D22 (deformed bar having 22mm in diameter) high strength bar having screw-type knot were used as longitudinal reinforcement. Three pieces of U8 (deformed high strength PC tendons having 8mm in diameter), which were arranged as stirrup in

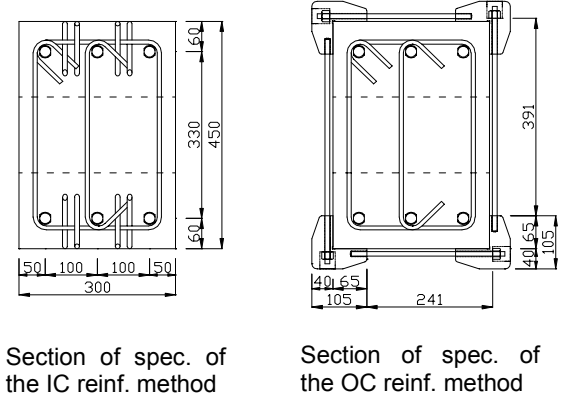


Fig. 1(b) Details of specimens (part 2)

100mm intervals, and round sectional unbonded-type prestress tendons 7.1Φ, 9.2Φ and 11Φ (having 7.1mm, 9.2mm and 11mm in diameter) were used as reinforcement for web opening.

On the IC reinforcing method specimens (except Spec. No.3), four pieces of unbonded-type prestress tendons were arranged symmetrically having 40 degree of inclined angle against longitudinal direction in inside of specimens.

On the OC reinforcing method specimen (Spec. No.3), a pair of prestress tendons was placed round on the surface of specimen near web opening. Those round tendons were connected each other by steel corner blocks, and same amount of prestressed tensile force was loaded to those tendons, not only tendons placed to beam-depth direction but also beam-width direction.

On all twelve specimens, prestressed tensile forces for tendons were loaded by the post tension-style loading. To normalize the effects of prestressed tensile

Table 1. List of specimens

Spec. No.	PS tendons' diameter, mm	Reinf. method	F_c MPa	p_p %	P_l kN	P_l/P_v	σ_D N/mm ²
No.1	7.1	IC	30	0.23	---	---	---
No.2					31	0.7	0.8
No.3	OC	0.27		40	0.5	0.8	
No.4				9.2	IC	0.38	15
No.5	31	0.4					0.8
No.6	61	0.8					1.7
No.7	11	IC	60	0.54	---	---	---
No.8					31	0.3	0.8
No.9					61	0.6	1.7
No.10			60	0.54	---	---	---
No.11					31	0.3	0.8
No.12					61	0.6	1.7

IC: IC reinforcing method OC: OC reinforcing method F_c : Designed concrete compressive strength

p_p : Amount of reinforcement for web opening calculated by diameter of prestress tendons arranged near web opening

P_l : Prestress tensile force per one piece of prestress tendon P_y : Yield tensile force of prestress tendon ($=\sigma_y A$)

σ_D : average prestress suffer to crack occurred near web opening which are assumed that crack angle against longitudinal direction is equal to 45 degree;

$$\sigma_D = n \cdot P_l \cdot \cos(\theta_p \cdot -\pi/4) / (b \cdot (\sqrt{2} \cdot D - H)) \square\square\square(l)$$

n : Number of prestress tendons which cross section having 45 degree of inclined angle

θ_p : Inclined angle of prestress tendons against longitudinal axis of specimens

Table 2. Mechanical properties of steel bars and concrete

Steel bar	$s\sigma_y, p\sigma_y$ N/mm ²	$s\sigma_t$ N/mm ²	E_s GPa	A mm ²	Concrete	σ_B MPa	$c\sigma_t$ MPa	E_c GPa
Longitudinal bars D22	1152	1279	190	387	For Spec. No.1-3	42	3.0	25
Stirrup U8	874	943	195	50	For Spec. No.4-6	39	2.8	24
PS Tendon for the IC method 7.1Φ	1182	1192	185	40	For Spec. No.7-9	35	2.6	24
PS Tendon for the IC method 9.2Φ	1244	1282	193	66	For Spec. No.10-12	66	3.8	29
PS Tendon for the OC method 9.2Φ	1273	1312	196	66				
PS Tendon for the IC method 11Φ	1220	1292	195	95				

$s\sigma_y, p\sigma_y$: Yield strength $s\sigma_t, c\sigma_t$: Tensile strength E_s, E_c : Elastic modulus A : Section area
 σ_B : Compressive strength

force, which may be varied by reinforcing method, the authors defined σ_D as a average prestress suffer to crack occurred near web opening which were assumed that crack angle against longitudinal direction was equal to 45 degree shown in **Fig. 2**.

On the IC reinforcing method specimens, specimens had 0.23%, 0.38% and 0.54% of ratio of reinforcement for web opening p_p calculated by diameter of prestress tendons arranged near web opening.

Fig. 4 shows loading setup. Specimens were suffered from multi-cyclic shear-bending force. Peak point of each loading cycle were decided by deformed angle of specimen R ; R were decided to $\pm 1/500\text{rad.}$, $\pm 1/200\text{rad.}$, $\pm 1/100\text{rad.}$, $\pm 1/67\text{rad.}$ and $\pm 1/50\text{rad.}$

To observe crack near web opening, the authors decided the area of 330mm×330mm near web opening, shown in **Fig. 4**, and width of crack occurred in that

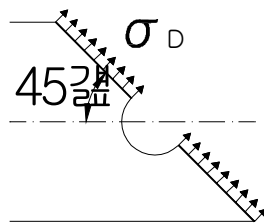
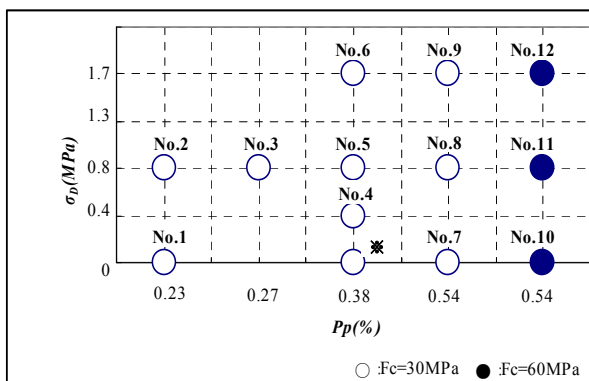


Fig. 2. Definition of σ_D



*: by Takasaki et. al. (2002) ¹⁾

Fig. 3. Relations between σ_D and p_p in each spec.

area were measured frequently. Crack width W was defined as width perpendicular to longitudinal direction of crack which occurred diagonally.

3. Result and discussion

3.1 General behavior

Experimental results are shown in **Table 3**, and relations between shear force Q and deformation angle R and crack pattern are shown in **Fig. 5**. On specimens of the IC reinforcing method width of crack occurred near web opening became wide and specimens became failure.

On spec. No.1 and No.2, unbonded-type tendons yielded and reached to maximum shear force just before R was equal to $+1/67$ rad. But on other specimens of the IC reinforcing method, shear force became maximum at the point that R was equal to $+1/67$ rad. On spec. No.3, specimen of the OC reinforcing method, shear failure were occurred on upper and lower area of web opening when R was equal to $-1/50$ rad., then Q had fallen.

Relations between shear crack strength near web opening τ_{sco} and average prestress σ_D are shown in **Fig. 6**. As figure shows, generally τ_{sco} increased according to growth of σ_D , that is growth of prestressed tensile

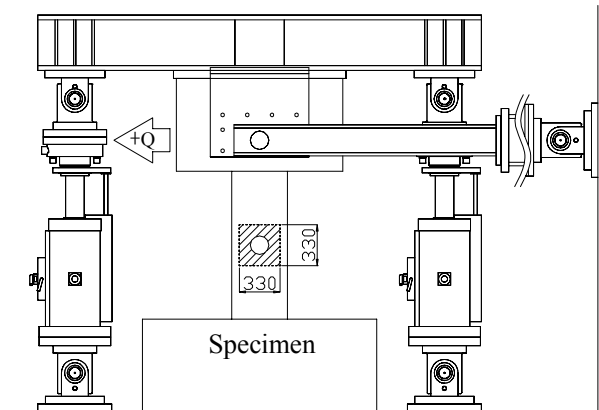


Fig. 4. Loading setup
(Hatched area shows crack-observing area)

Table 3. Experimental result

Spec. No.	eQ_{bu} kN	eQ_{AL} kN	eQ_{AS} kN	eQ_{suo} kN	eQ_{sco} kN	eQ_{max} kN	eQ_{max}/eQ_{suo}
No.1	783	179	346	294	66	343	1.17
No.2					101	356	1.21
No.3				308	122	475	1.54
No.4		175	339	325	55	387	1.19
No.5					106	401	1.23
No.6					129	449	1.38
No.7		168	329	347	58	408	1.18
No.8					113	422	1.22
No.9					141	424	1.22
No.10		218	403	395	63	478	1.21
No.11					98	515	1.31
No.12					138	507	1.29

eQ_{bu} : Calculated maximum bending strength eQ_{AL} : Allowable shear strength for sustained loading by AIJ approved formula for reinforced concrete beam not having web opening eQ_{AS} : Allowable shear strength for temporary loading by AIJ approved formula for reinforced concrete beam not having web opening eQ_{suo} : Shear strength calculated by the Hirosawa's formula ²⁾:

$$eQ_{suo} = \left\{ \frac{0.092 \cdot k_u \cdot k_p \cdot (F_c + 18)}{M/Q \cdot d + 0.12} \cdot \left(1 - \frac{1.61 \cdot H}{D} \right) + 0.85 \cdot \sqrt{p_s \cdot \sigma_y + p_p \cdot \sigma_y} \right\} \cdot b \cdot j \quad (2)$$

p_p : Ratio of reinforcement for web opening by prestress tendons arranged near web opening:

$$p_p = \sum (a_s \cdot (\sin \theta_p + \cos \theta_p)) / b \cdot c \quad (3)$$

a_s : Sectional area of prestress tendons θ_p : Inclined angle of prestress tendons against longitudinal axis of specimens

$p \sigma_y$: Yield strength of prestress tendon eQ_{sco} : Shear cracking load near web opening taken in experiments

eQ_{max} : Maximum shear strength taken in experiments

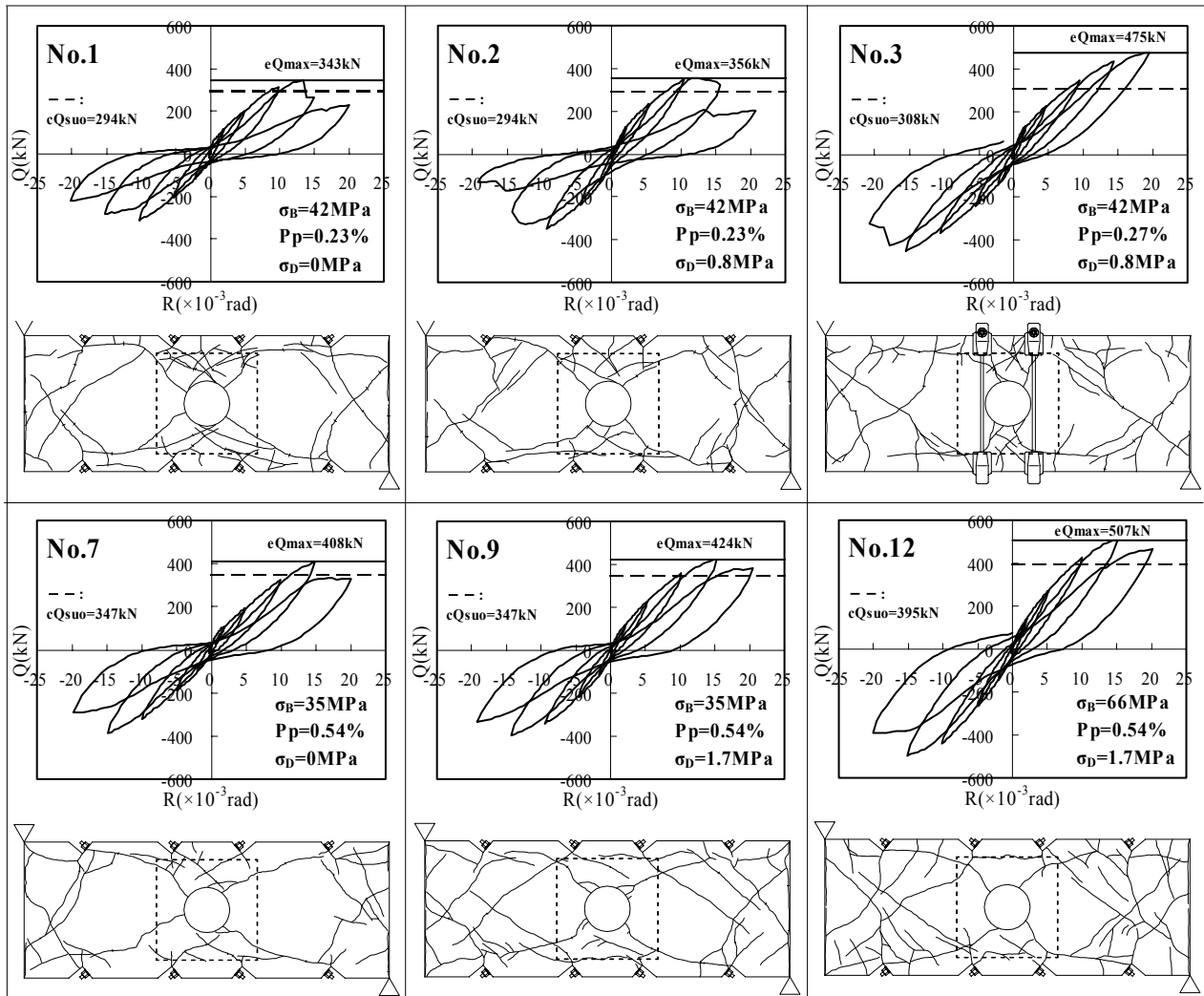


Fig. 5. Q-R relations and crack patterns

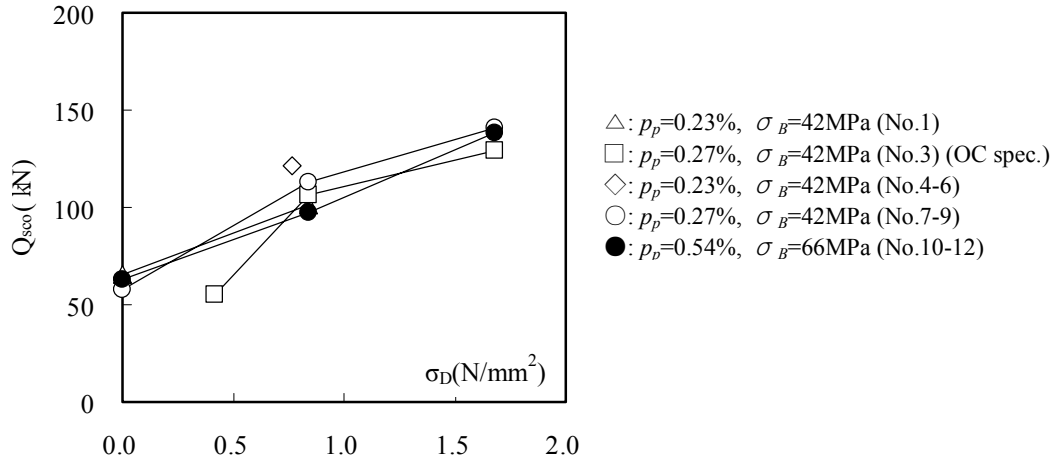


Fig. 6. Q_{sco} - σ_D relation

force. And figure shows that τ_{sco} may not be influenced by p_p and concrete compressive strength σ_B .

After shear crack occurred, number of shear crack near web opening became smaller according to growth of σ_D , and other shear cracks were occurred outside of web opening area.

3.2 Crack width near web opening

Relations between average shear stress τ and maximum crack width W_{max} are shown in Fig. 7, 8, 9 and Fig. 10. On those figures range of τ were positive. As figure shows, crack width, both width when shear loading and unloading, became small according to growth of σ_D and p_p . For example, if σ_D is equal to 1.7MPa, that is to say that σ_D is a small value, crack width became about 1/20 as small as that if σ_D is equal to zero, when specimen is under situation of allowable shear stress for long sustained loading. And as figure shows, difference of concrete compressive strength may not be influenced to relations between prestressed tensile force and W_{max} . As compared between spec. No.2 and No.3, strength when shear crack occurred were approximately equal, but crack width of spec. No.3 was smaller than that of spec. No.2. Reason why that may be that stiffness for axial direction of prestress tendon used for spec. No.3 was larger than that of spec. No.2, because tendon for spec. No.3 had larger area of section and shorter length than those of spec. No.2.

Relations between σ_D and crack width when specimens were under situation of allowable shear stress for long sustained loading $_{AL}W_{max}$ are shown in Fig. 11.

Generally in Japan, maximum crack width when prestressed concrete structures are under situation of allowable shear stress for long sustained loading may be recommended to 0.2mm or below. But as shown in figure, crack width $_{AL}W_{max}$ of specimens which had no prestressed tensile force for prestress tendons reached between 1.10mm and 1.31mm. Those values are approximately six times as large as 0.2mm. So those

facts show that it is estimated that maximum crack width of normal reinforced concrete beam with web opening, not having the IC or OC reinforcing method, may be exceed to 0.2mm, when beam is under situation of allowable shear stress for long sustained loading.

3.3 Behavior of reinforcing bar near web opening

Relations between shear force Q and vertical shear force loaded by prestress tendons V_p , vertical shear force loaded by stirrup arranged near web opening V_s are shown in between Fig. 13 to Fig. 18. As figure shows, point that shear force loaded by stirrup arranged near web opening became to grow may appear later according to growth of prestressed tensile force. That is because large prestressed tensile force prevents to become width of shear crack near web opening wide.

Among specimens not having prestressed tensile force, characteristics between σ_D and V_p , V_s are similar to each other even if concrete compressive strength may vary. But on specimen having σ_B is equal to 66MPa and having prestressed tensile force, shear force not only loaded by prestress tendon but also loaded by stirrup arranged near web opening tend to grow. So it became clear that prestressed tensile force makes prestress tendon and stirrup arranged near web opening act effectively.

3.4 Shear strength

In Japan the formula suggested by Dr. Hirose, which is shown in Eq.(1)²⁾, is well-known for calculation of shear strength of reinforced concrete beam with web opening. Exactly that formula is not following to beam with the OC and IC reinforcing method, but in this section the authors tried to apply that formula to each specimen. Relation between shear strength calculated by the Hirose's formula $_{e}Q_{suo}$ and that taken in experiments $_{e}Q_{max}$ are shown in Fig. 19. In this figure, it is assumed that prestress tendons were considered as reinforcement for web opening like

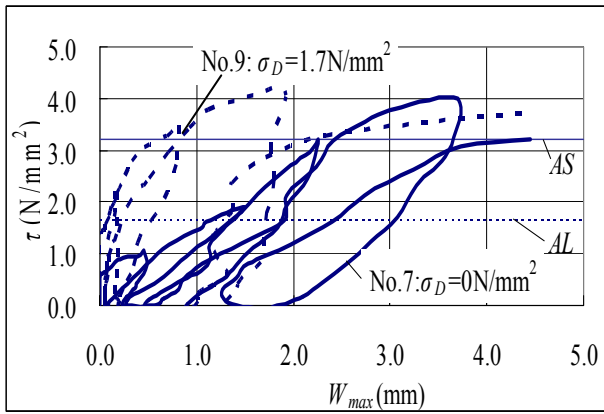
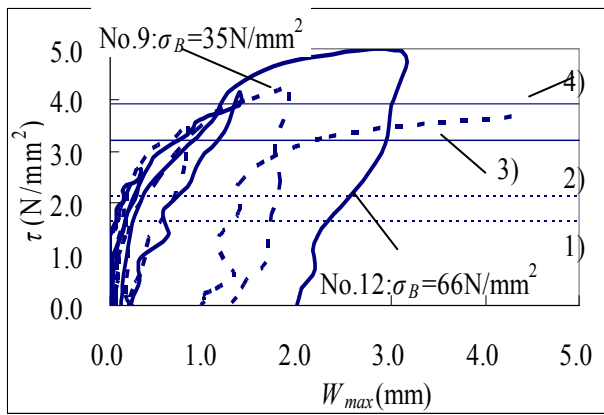
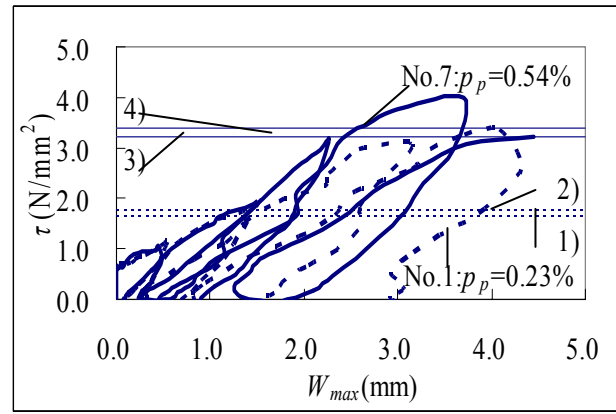


Fig. 7. τ - W_{\max} relation (spec. selected by the scope of variation of σ_D)



Dotted line 1): AL for No.9
Dotted line 2): AL for No.12
Line 3): AS for No.9
Line 4): AS for No.12

Fig. 9. τ - W_{\max} relation (spec. selected by the scope of variation of σ_B)



Dotted line 1): AL for No.7
Dotted line 2): AL for No.1
Line 3): AS for No.7
Line 4): AS for No.7

Fig. 8. τ - W_{\max} relation (spec. selected by the scope of variation of p_p)

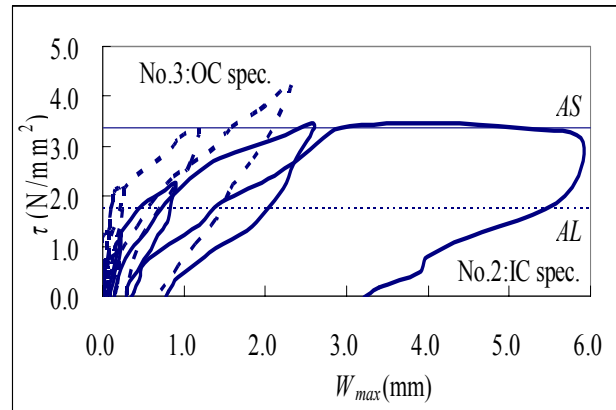


Fig. 10. τ - W_{\max} relation (spec. selected by the scope of reinforcing method)

From Fig. 7 to Fig. 10;

$$\tau = Q/(b \cdot j)$$

AL: Allowable shear stress for long sustained loading $AL = cQ_{AL}/(b \cdot j)$

AS: Allowable shear stress for temporary loading $AS = cQ_{AS}/(b \cdot j)$

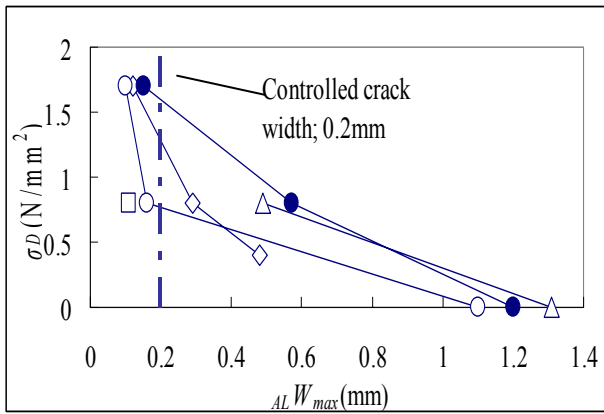
stirrup. As figure shows, eQ_{\max} exceeded cQ_{suo} . Value of eQ_{\max}/cQ_{suo} , that is safety factor, became between 1.14 and 1.38 on specimens with the IC reinforcing method, and became 1.54 on specimen with the OC reinforcing method.

Safety factor for specimen with the OC reinforcing method was larger than that for specimens with the IC reinforcing method. It may be thought that the reason above is that prestress tendon for the OC reinforcing method has larger effects for shear strength than that of the IC reinforcing method.

4. Conclusion

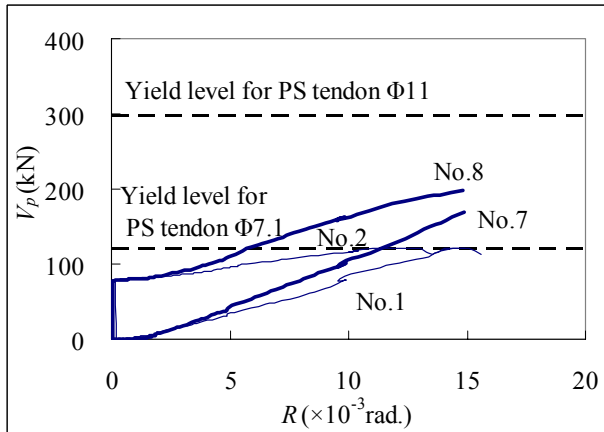
This study examined the effects of new reinforcing method of reinforced concrete beam with web opening. The following conclusion can be made:

1. Shear crack strength near web opening increases according to growth of prestressed tensile force for prestress tendons arranged or placed near web opening.
2. Crack width become small according to growth of prestressed tensile force for prestress tendons arranged or placed near web opening. Moderate prestressed tensile force for prestress tendons and



- \triangle : $p_p=0.23\%$, $\sigma_B=42\text{MPa}$ (No.1-2)
 \square : $p_p=0.27\%$, $\sigma_B=42\text{MPa}$ (No.3) (OC spec.)
 \diamond : $p_p=0.23\%$, $\sigma_B=42\text{MPa}$ (No.4-6)
 \circ : $p_p=0.27\%$, $\sigma_B=42\text{MPa}$ (No.7-9)
 \bullet : $p_p=0.54\%$, $\sigma_B=66\text{MPa}$ (No.10-12)

Fig. 11. σ_D - $AL W_{\max}$ relation



- No.1: $\Phi 7.1$, $\sigma_D=0$
 No.2: $\Phi 7.1$, $\sigma_D=0.8\text{N/mm}^2$
 No.7: $\Phi 11$, $\sigma_D=0$
 No.8: $\Phi 11$, $\sigma_D=0.8\text{N/mm}^2$

Fig. 13. V_p - R relation (spec. selected by the scope of variation of p_p and σ_D)

amount of reinforcement for web opening may lead crack width under controlled width value.

- If specimens are made of high strength concrete, prestressed tendon and stirrup arranged near web opening act effectively.
- The Hirose's formula, which is famous in Japan to estimate shear strength of reinforced concrete beam with web opening, is applicable to specimens having the IC and OC reinforcing methods.

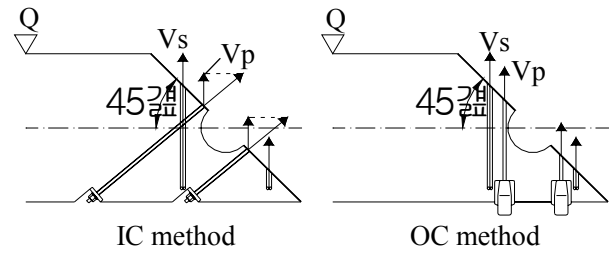
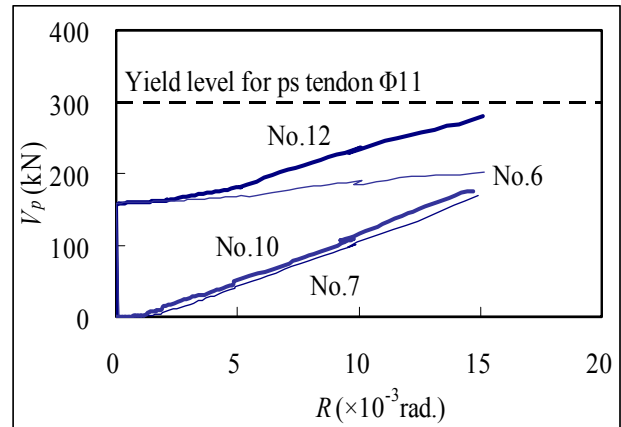
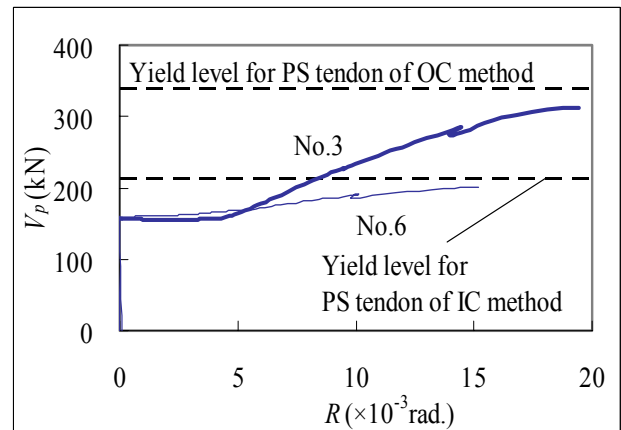


Fig. 12. Shear force at PS tendons and stirrup



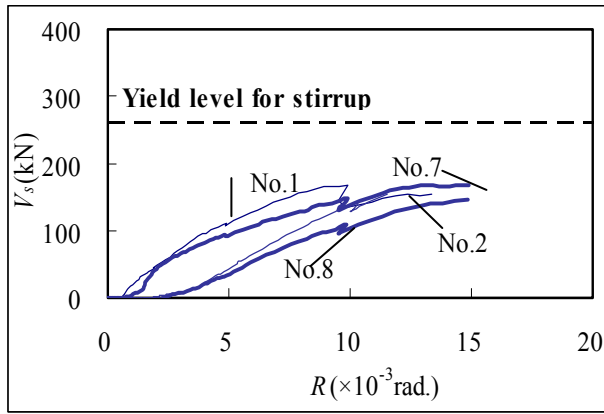
- No.6: $\sigma_B=35\text{MPa}$, $\sigma_D=1.7\text{N/mm}^2$
 No.7: $\sigma_B=35\text{MPa}$, $\sigma_D=0$
 No.10: $\sigma_B=66\text{MPa}$, $\sigma_D=0$
 No.12: $\sigma_B=66\text{MPa}$, $\sigma_D=1.7\text{N/mm}^2$

Fig. 14. V_p - R relation (spec. selected by the scope of variation of σ_B and σ_D)

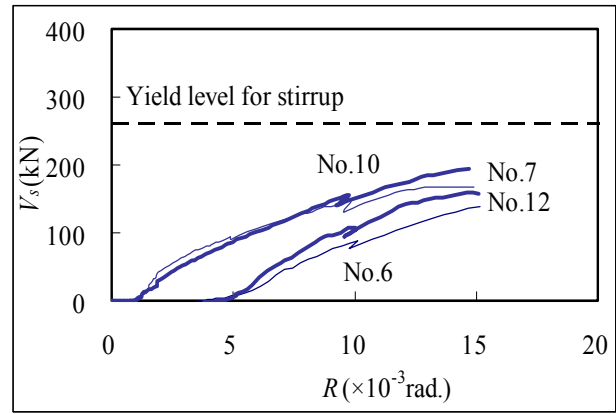


- No.3: OC method
 No.6: IC method

Fig. 15. V_p - R relation (spec. selected by the scope of variation of reinforcing method)



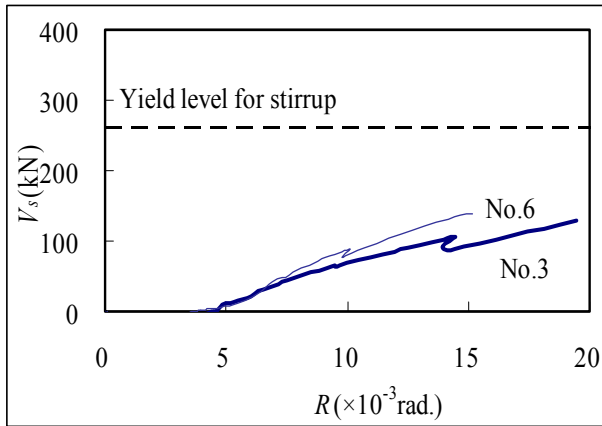
No.1: $\Phi 7.1$, $\sigma_D=0$
 No.2: $\Phi 7.1$, $\sigma_D=0.8\text{N/mm}^2$
 No.7: $\Phi 11$, $\sigma_D=0$
 No.8: $\Phi 11$, $\sigma_D=0.8\text{N/mm}^2$



No.6: $\sigma_B=35\text{MPa}$, $\sigma_D=1.7\text{N/mm}^2$
 No.7: $\sigma_B=35\text{MPa}$, $\sigma_D=0$
 No.10: $\sigma_B=66\text{MPa}$, $\sigma_D=0$
 No.12: $\sigma_B=66\text{MPa}$, 1.7N/mm^2

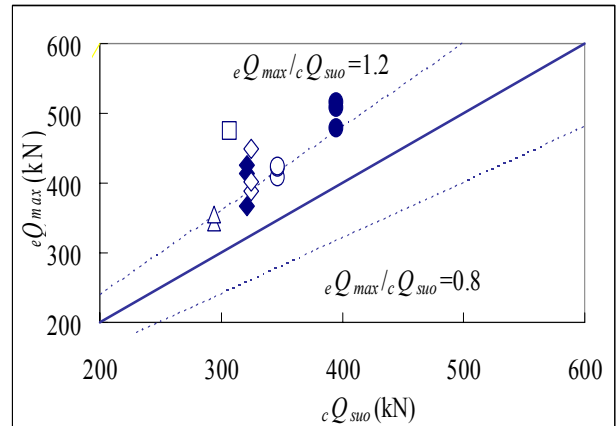
Fig. 16. V_s - R relation (spec. selected by the scope of variation of p_p and σ_D)

Fig. 17. V_s - R relation (spec. selected by the scope of variable of σ_B and σ_D)



No.3: OC method
 No.6: IC method

Fig. 18. V_s - R relation (spec. selected by the scope of variation of reinforcing method)



\triangle : No.1-2 \square : No.3 (OC spec.)
 \diamond : No.4-6 \circ : No.7-9 \bullet : No.10-12
 \blacklozenge : by Takasaki et. al. ¹⁾

Fig. 19. eQ_{\max} - cQ_{suo} relation

Acknowledgement

The authors thank the Neturen Co., Ltd. for their fund and giving PS tendons. The authors also thank Mr. Yuta Takasaki, staff of the Building Center of Japan, for his advice on experiments.

References

- 1) Takasaki, Y., Katori, K. and Hayashi, S. (2002), Study on Shear Reinforcement for RC Beam with Web Opening under Consideration of Crack Control, Proceedings of the Japan Concrete Institute, Vol. 24, No.2, pp.295-300
- 2) Architectural Institute of Japan (1987), Data for Ultimate Strength Design of Reinforced Concrete Structures, pp.40-42
- 3) Akagi, D., Yanase, T., Katori, K. and Hayashi, S. (2003), Experimental Study on Shear Behavior of Prestressed Reinforced Concrete Beams with Web Openings. Proceedings of the Japan Concrete Institute, Vol. 25, No.2, pp.409-414
- 4) Architectural Institute of Japan (1999), AIJ Standard for Structural Calculation of Reinforced Concrete structure
- 5) Architectural Institute of Japan (1986), Recommendations for Design and Construction of Partially Prestressed Concrete (Class 3 of Prestressed Concrete) Structures
- 6) Yanase, K., Ohno, Y., Li, Z. and Minami, H. (2002), Shear Crack Width of Reinforced Concrete Beams, Proceedings of the Japan Concrete Institute, Vol. 24, No.2, pp.343-348