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Diagrid Structural System for High-Rise Buildings: Applications of a Simple Stiffness-based Optimized Design

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Abstract

The ingenuity of structural engineers in the field of tall and super-tall buildings has led to some of the most remarkable inventions. During this evolution of structural engineering concepts in the last 100 years, the technical challenges that engineers encountered were extraordinary and the advances were unprecedented. However, as the accomplishments of structural engineers are progressing, the desire for taller and safer structures is also increasing. The diagrid structural system is part of this evolving process as it develops a new paradigm for tall building design combining engineering efficiency and new architectural expression. The first appearances of this type of tall buildings have already been constructed and the interest of both engineering and architectural communities is growing mainly due to the many advantages compared to other structural systems. This paper presents a simple approach on optimizing member sizes for the diagonals of steel diagrid tall buildings. The optimizing method is based on minimizing the volume of the diagonal elements of a diagrid structure. The constraints are coming from the stiffness-based design, limiting the tip deflection of the building to widely accepted regulative limits. In addition, the current paper attempts to open the discussion on the important topic of optimization and robustness for tall buildings and also studies the future of the diagrid structural system.

Keywords: Diagrid structures, High-rise buildings, Optimization, Stiffness-based design, Robustness

1. Introduction

The evolution of structural system concepts for tall buildings has been driven by the increasing need to achieve greater heights. For more than 100 years, structural engineers have been able to design and construct buildings which have risen higher and higher. This continuous process involved many outstanding advances and numerous new and innovative structural systems. Starting from the typical steel or reinforced concrete rigid frames in the late nineteenth century which could reach 20-30 stories high, today's technological advances allow engineers to build structures more than 100 stories high using pioneering structural systems.

In this process, many new ideas have emerged among which the relatively new and groundbreaking high-rise diagrid structural system. The appearance of the diagrid system occurred as a result of the architectural appreciation of the aesthetic potential of diagonal members which started with braced tube structures. The expansion of this concept to a system without vertical columns led to the birth of the diagrid structure. The benefits of placing diagonal members on the perimeter of the building are many,

but certainly the most important one is that the efficiency of the system is far greater than of a system where the lateral bearing structure is confined in the narrow core. For these two reasons, diagrid structures have attracted the interest of engineers and architects and are increasingly used as a tall building structural system. The most well-known examples are the Hearst Headquarters in New York City, the Swiss Re Building in London both by Sir Norman Foster and the Guangzhou Twin towers in Guangzhou China by Wilkinson Eyre.

From a structural standpoint, the research interest in diagrid tall buildings has also increased in the last two decades. One of the first papers in the field studied the characteristics of tall diagrid buildings and presented a new methodology for preliminary design (Moon, 2007). In this paper, a member sizing methodology is presented considering the impact of the diagonal angle on the behavior of the diagrid system under wind loading. The proposed method in this paper is a stiffness-based member sizing methodology which can be applied to a typical height range of diagrid structures. Although constructability is highlighted as a major factor in the design process in that paper, this issue is further examined in a following paper in which node construction and façade construction are described as the key factors (Moon, 2009).

The cyclic behavior of diagrid nodes has been studied in detail focusing on hysteresis characteristics, welding

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methods and failure modes (Kim et al., 2010). Topology optimization has also been applied for the extraction of diagonal members in a diagrid structural system for tall buildings (Lee et al., 2010), while the concept of varying diagonals angles has been explored in an attempt to reach more efficient designs (Zhang et al., 2012; Zhao, 2015). Based on the diagrid, different concepts have been developed as well, such as the hexagrid system for tall buildings (Nejad, 2011) which have numerous architectural advantages. A very useful overview of the structural behavior of tall diagrid structures is presented in a series of papers (Mele et al., 2014a; Mele et al., 2014b).

The current paper is applying a simple stiffness-based optimization algorithm which can be used for the preliminary design of tall diagrid structures. The governing factor for the optimization procedure is the tip displacement of the building which is set at H/500, where H is the total height of the structure. Then, following an iterative procedure which is described in detail in the following section, the algorithm applies the optimized member sizing technique. The proposed optimization procedure is based on a virtual work approach and it is applied to three typical tall building diagrid geometries, a 48-story, a 60-story and a 72-story building.

2. Optimization Procedure for Diagrid High-Rise Buildings

This section includes the theoretical background for the optimization procedure which will be followed in this paper. For the proposed method, the core gravity framing will not be included in the calculations and therefore its participation in the lateral stiffening system will not be considered during the optimization process. It is assumed that the lateral stiffness of the buildings comes only from the diagrid system. The stiffness-based optimization procedure presented in this paper builds on previous work (Callow, 2001; Thornton et al., 1990) and develops a virtual work optimization procedure for diagrid structures with circular pipe sections as the diagonals.

Based on the general consideration of the principle of virtual work, the displacement of a given point T in a structure under any external load is given by the following expression:

$$
\delta_{T,\alpha} = \Sigma \frac{(N_i)_{x,1} \cdot (N_i)_{x,\alpha}}{EA} dL + \Sigma \frac{(N_i)_{y,1} \cdot (N_i)_{y,\alpha}}{EA} dL
$$
$$
+ \Sigma \frac{(N_i)_{z,1} \cdot (N_i)_{z,\alpha}}{EA} dL + \Sigma \frac{(M_i)_{x,1} \cdot (M_i)_{x,\alpha}}{EI} dL
$$
$$
+ \Sigma \frac{(M_i)_{y,1} \cdot (M_i)_{y,\alpha}}{EI} dL + \Sigma \frac{(M_i)_{z,1} \cdot (M_i)_{z,\alpha}}{EI} dL \qquad (1)
$$

Where $\delta_{T,a}$ is the displacement at the point of interest T, due to the loading a, $(N_i)_{x,1}$ is the axial force of member i due to the unit loading at point T, $(N_i)_{y,1}$ is the y shear force of member i due to the unit loading at point T,

$(N_i)_{z,1}$ is the z shear force of member i due to the unit loading at point T, $(M_i)_{x,1}$ is the torsional moment of member i due to the unit loading at point T, $(M_i)_{y,1}$ is the bending moment around y-y axis of member i due to the unit loading at point T, $(M_i)_{z,1}$ is the bending moment around z-z axis of member i due to the unit loading at point T, $(N_i)_{x,a}$ is the axial force of member i due to the external loading a, $(N_i)_{y,a}$ is the y shear force of member i due to the external loading a, $(N_i)_{z,a}$ is the z shear force of member i due to the external loading a, $(M_i)_{x,a}$ is the torsional moment of member i due to the external loading a, $(M_i)_{y,a}$ is the bending moment around y-y axis of member i due to the external loading a and $(M_i)_{z,a}$ is the bending moment around z-z axis of member i due to the external loading a.

The proposed method will use the general expression for the displacement of a point under a given load and apply it for the member sizing optimization technique of a tall diagrid building structure. For the specific purposes of a diagrid structure, the shear forces and torsional moment contributions can be ignored due to the small relative contribution compared to the rest of the contributions. In most cases, the diagrid structural system can be considered in such a way that the diagonals work mostly in axial forces. However, for the proposed work, the two bending moments in the y and z axis will also be considered. Therefore the expression for the deflection of point T under a given load a in Eq. (1) becomes:

$$
\delta_{T,\alpha} = \Sigma \frac{(N_i)_{x,1} \cdot (N_i)_{x,\alpha}}{EA} dL
$$
$$
+ \Sigma \frac{(M_i)_{y,1} \cdot (M_i)_{y,\alpha}}{EI} dL + \Sigma \frac{(M_i)_{z,1} \cdot (M_i)_{z,\alpha}}{EI} dL \qquad (2)
$$

For a steel pipe, the cross sectional area A and the moment of inertia I are given by the following expressions:

$$
A = \frac{\pi(D^2 - d^2)}{4} \qquad (3)
$$

$$
I = \frac{\pi(D^4 - d^4)}{64} = \frac{A}{16}(D^2 + d^2) \qquad (4)
$$

Where D is the external diameter of the steel pipe and d is the internal diameter of the steel pipe.

It is well known, that the deflection of a point in a structure can be calculated using Eq. (1), by calculating the influence of every structural member to the point's deflection. Therefore, if a particular member in the structure is increased (either in cross section or moment of inertia) the influence of the particular member to the point's deflection increases.

The proposed optimization technique is aiming at minimizing the volume of the structure and therefore the deflection influence efficiency X_i of every member i can be expressed by the following equation (Callow, 2001):

$$X_i$$

$$= \frac{\dfrac{(N_i)_{x,1} \cdot (N_i)_{x,\alpha} L_i}{EA}}{A_i L_i} + \frac{\dfrac{(M_i)_{x,1} \cdot (M_i)_{x,\alpha} L_i}{EI}}{A_i L_i} + \frac{\dfrac{(M_i)_{x,1} \cdot (M_i)_{x,\alpha} L_i}{EI}}{A_i L_i}$$

$$= \frac{(N_i)_{x,1} \cdot (N_i)_{x,\alpha}}{E(A_i)^2} + \frac{(M_i)_{x,1} \cdot (M_i)_{x,\alpha}}{EA_i I_i} + \frac{(M_i)_{y,1} \cdot (M_i)_{y,\alpha}}{EA_i I_i} \qquad (5)$$

Generally speaking in terms of the mathematical aspect of the proposed optimization technique, the objective function regards minimizing the volume of the structure, subjected to the equilibrium of the structural system and the condition of H/500 for the tip displacement of the building.

Using the Eqs. (3), (4) and (5), the following expression is reached for the deflection influence efficiency factor of each diagonal pipe member i:

$$X_i = \frac{16}{E\pi^2 (D^2 - d^2)^2} \cdot \{(N_i)_{x,1} \cdot (N_i)_{x,\alpha}$$

$$+ \frac{16[(M_i)_{x,1} \cdot (M_i)_{x,\alpha} + (M_i)_{y,1} \cdot (M_i)_{y,\alpha}]}{(D^2 + d^2)}\} \qquad (6)$$

2.1. The Iterative Process

The optimization process for the present paper follows a slightly modified procedure of the one described in previous work (Callow, 2001).

The first step of the iterative process is to create a simple linear FEM model of the structure and analyze it for two loading conditions: one for the actual acting wind loading condition for which the design needs to be met and one for a unit loading condition which includes only one unit lateral load at the tip of the building. For the particular case of diagrid structures, this model needs to include the diagrid diagonals and the slab of each floor in order to achieve a 3D behavior and consider the lateral constraints of each diagonal and every floor. As mentioned before, the gravity core is not included in this method. The models are created using minimal arbitrary cross sections for the diagonal diagrid members and common slab thickness. It must be noted here that a good starting point for the optimization process would be to perform an initial gravity-induced (dead and live load) design and consider the outcome of this design as the starting point for the diagonal cross sections. For the work presented in the current paper, arbitrary small sections are accounted as the initial point for the optimization process. Another important point is that the results of the first analysis in this case would reveal that the tip deflection of the building is extremely high under wind, since the cross sections of the diagonals are very small.

The second step of the process requires the extraction of the axial forces and two bending moments of each diagonal diagrid member due to the two loading conditions (wind and unit load). Having the forces and moments of all the members, a simple virtual work calculation for the tip deflection δ_{VW} is performed. For the current method, this calculation is not expected to match the FEM result of the wind analysis of the first step, since the effect of the slabs is not taken into account. Nevertheless, this calculation provides the necessary information on the influence of the diagrid diagonal members to the tip deflection of the structure under wind.

As a third step, the deflection influence efficiencies X_i of each member i are calculated and a list of the members is created following a descending order of the X_i values. Therefore, the member with the highest deflection efficiency factor is easily found at the top of the list.

Next, this member's area is increased (by a finite amount defined by the designer) which results firstly to the decreasing of the deflection δ_{VW} and secondly to this member's X_i value decrease. This decrease in the X_i value would most often lead to the member dropping down in the list and another member having the maximum X_i value.

Again, the new member is increased and this process is repeated until the deflection δ_{VW} is lower than the allowable limit. One very important point is that since the slab contribution is not considered in the calculation of δ_{VW}, the target displacement of the optimization process needs to be lower than the actual allowable limit which is H/500.

Finally, the actual target displacement in the optimization process is found through additional iterations at the end of the process and validations with the FEM analysis.

3. Design Studies

3.1. The Three Buildings

The optimization procedure is applied to three diagrid building geometries, a 48-story building [A], a 60-story building [B] and a 72-story building [C]. Two different values for the angle θ of the diagonal members were chosen; namely 63° and 69°. These angles were chosen based on previous work done by the authors (Moon, 2007; Moon, 2008) and Figs. 1 and 3 show the corresponding geometries of the buildings analyzed. The typical module in the case where $\theta = 63°$ spans 6 floors and has a height of 24 m, while the typical module in the case where $\theta = 69°$ spans 8 floors and has a height of 32 m (Figs. 2 and 4 respectively). The 6 buildings share some geometrical characteristics, such as the story height which is 4 m and the square floorplan dimensions which are 36 m × 36 m. All FEM simulations were carried out using the finite element software ABAQUS (Simulia, 2015). All diagrid members were modelled using pipe elements and the slab was simulated using shell elements. The base nodes of all buildings were assumed fixed and the buildings were assumed to have a linear elastic behavior. The wind loading for the design of the buildings is calculated according
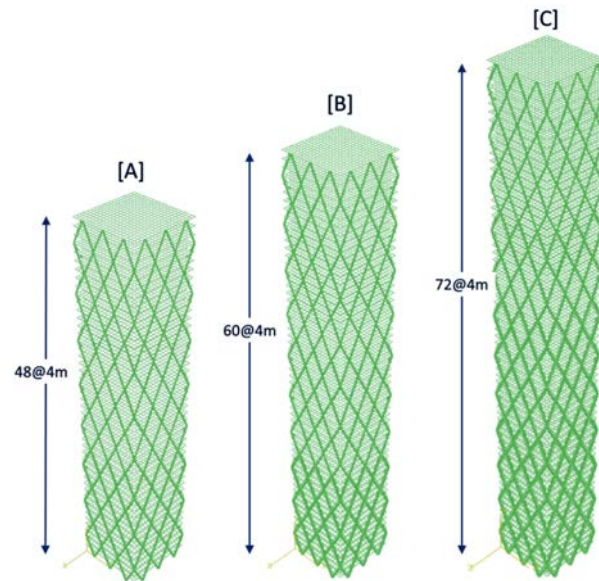
Figure 1. Building Geometries for θ=63°.
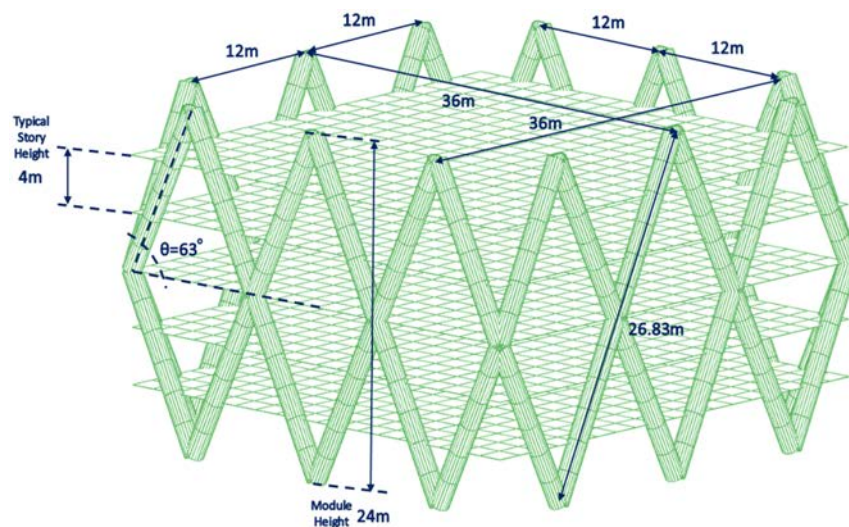
Figure 2. Typical Module Geometry for θ=63°.

to ASCE 7-05. The buildings are in New York, with occupancy category III, exposure category B and damping ratio 1%. The basic wind speed is set at 103 mph according to the ATC (ATC).

3.2. Application of the Optimization Process - Results

For the purposes of the application of the optimization process to the six buildings, the initial cross sections of the diagrid diagonals are chosen as arbitrarily very small. The diameters of the circular hollow sections were predetermined as constant (Table 1). A common value of the diameter was assigned in certain groups of modules. This

approach was adopted for constructability reasons, since the connection of diagonals sharing the same diameter reduces both the construction time and cost. The initial thickness of every diagonal in the building had the arbitrarily small value of 5 mm as a starting point. At the end of each iteration the thickness of the critical member (the one with the highest deflection efficiency factor) was increased per 1mm and the calculation of the maximum displacement was repeated. The optimization process was completed when the tip deflection was lower than the maximum allowable limit. Table 2 presents the results regarding the total steel tonnage, the maximum thickness
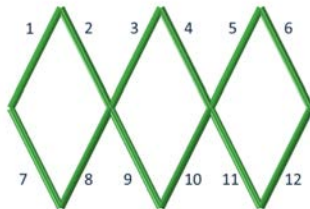
Figure 3. Building Geometries for θ=69°.

Figure 4. Typical Module Geometry for θ=69°.

obtained and the maximum displacement for each optimized building.

A first observation of the results of Table 2, leads to the conclusion that the angle θ = 69° is much more efficient and economic for all the diagrid geometries, compared to the angle θ = 63°. For the 72-, 60- and 48-story buildings the total tonnage is reduced 20%, 17.5% and 9% respectively when the angle θ = 69° is employed. Apparently, 69°

angle is more effective for the taller buildings. This finding comes in agreement with previous results by the authors (Moon 2008), where the angle of 69° was found to be the optimal for the 60- and 70-story buildings analyzed. Smaller maximum thicknesses are also obtained and the tip horizontal displacement limit is satisfied in all cases. Since all thicknesses had the initial value of 5 mm and the optimization process resulted in different values for each

Table 1. Diagonal Member Diameters (m)

Floor	θ = 63°			Floor	θ = 69°		
	48	60	72		48	60	72
1st - 6th	1	1.5	2	1st - 8th	1.1	1.5	1.9
7th - 12th	1	1.5	2	9th - 16th	1.1	1.5	1.9
13th - 18th	1	1.5	2	17th - 24th	0.9	1.1	1.9
19th -24th	0.7	0.9	2	25th - 32nd	0.9	1.1	1.6
25th - 30th	0.7	0.9	1.5	33rd - 40th	0.6	0.9	1.6
31st - 36th	0.7	0.9	1.5	41st - 48th	0.6	0.9	1.2
37th - 42nd	0.6	0.7	1.5	49th - 56th	-	0.7	1.2
43rd - 48th	0.6	0.7	1.1	57th – 60th	-	0.7	0.9
49th - 54th	-	0.7	1.1	61st – 64th	-	-	0.9
55th - 60th	-	0.7	0.7	65th – 72nd	-	-	0.9
61st - 66th	-	-	0.7				
67th - 72nd	-	-	0.7				

Table 2. Total steel tonnage, maximum thickness and displacement for the optimized buildings

	θ = 63°			θ = 69°		
	Tonnage (m³)	Max 't' (m)	Displacement (m) (obtained / limit)	Tonnage (m³)	Max 't' (m)	Displacement (m) (obtained / limit)
48-story	4037	0.075	0.375 / 0.384	3684	0.062	0.369 / 0.384
60-story	11017	0.13	0.474 / 0.48	9086	0.095	0.472 / 0.48
72-story	25372	0.169	0.575 / 0.576	20226	0.144	0.558 / 0.576

Figure 5. Diagonal notation.

diagonal, an interesting observation was made regarding the additional steel demand. Namely, the same general pattern was observed among all building geometries. At the bottom and middle part of each structure, the steel demand was greater in diagonals 7 and 12, while at the upper part of the structure the maximum steel demand was in diagonals 8 and 11 (notation in Fig. 5). Since the structure had an initially symmetric geometry and the wind loading was symmetric, the final geometry of the optimized buildings was also symmetric.

3.3. The Effect of Intermediate Loading Steps

The optimization process was done based on the initially extracted section forces; namely, those which correspond to the building with the arbitrarily small thicknesses. After a finite amount of iterations, the changes in members' sizes are expected to result in a different stiffness matrix and therefore in a different load distribution. As an outcome, the final thicknesses of the members are

expected to be slightly different. Fig. 6 shows the axial forces of the members located in the diagrid face where the wind is applied, in the 72-story building and 63° angle. This graph shows that as material is added to the diagonals, the force distribution changes and this will in turn result into a different thickness demand. As more intermediate steps are included into the optimization process, the procedure gains in accuracy.

3.4. The Effect of the Internal Core

In the current study the diagrid system is assumed to provide the whole lateral resistance to the building. Such an assumption is considered slightly conservative, since the internal core will also contribute to the horizontal stiffness (Moon, 2007). The core consideration is expected to yield smaller values for the thicknesses of the diagrid diagonal members.

3.5. The Effect of the Member Grouping Based on the Thickness

The current form of the optimization method results into a different thickness for each diagonal. Since many different member sizes are used, the construction cost will increase and this is why a common value for thickness per module should be applied during construction. This thickness value must satisfy the tip deflection limit and at the same time not increase significantly the total tonnage. The assessment of this value is considered the next step for the enhancement of the method.
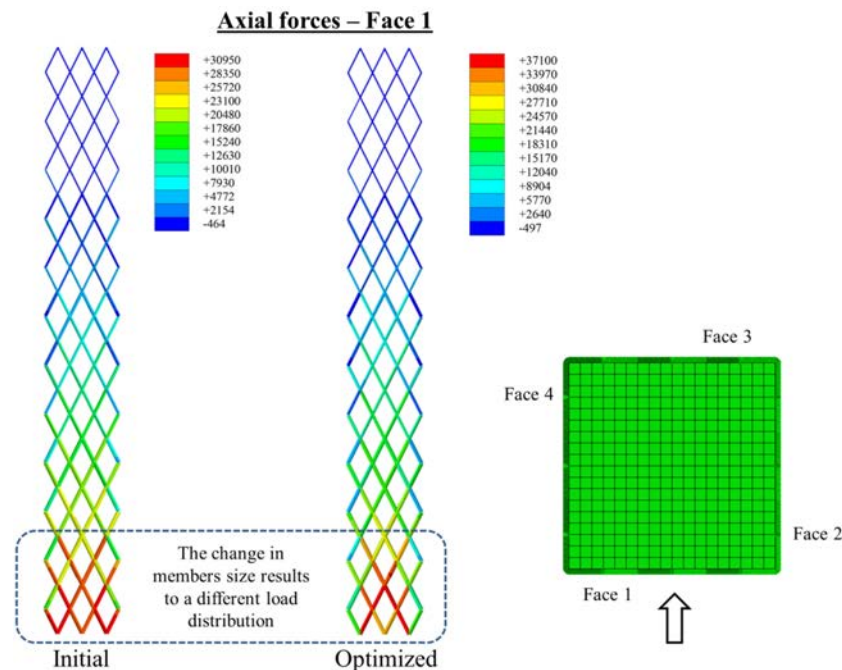
Figure 6. Axial forces in Face 1 of the 72-story building: Initial and Optimized model.

4. Optimization and Robustness

Among the numerous important aspects in tall building design, one has attracted increasing attention in the last decades. This aspect involves the response of tall buildings to extreme damaging events either natural or man-made. This whole field of research has received a lot of interest from researchers and designers in the last couple of decades mainly after the catastrophic collapse of the World Trade Center in New York in 2001 due to airplane impacts and the Alfred P. Murrah Building in Oklahoma due to an external explosion event. This structural property is particularly important for tall buildings more than short structures for two main reasons. Firstly, a tall and landmark building is considered by many as a more probable target for man-made damaging extreme events and secondly, a potential collapse of a tall building can lead to immense multi-level losses as observed during and after the collapse of the WTC.

For that reason, the theory of progressive collapse has been developed helping designers and researchers assess the capacity of a structure against an extreme event. Relevant research efforts aim at quantifying and ultimately minimizing structural vulnerabilities to a wide range of triggering events. In that sense, local damage is acceptable in a structural system as long as it does not jeopardize the overall structural integrity of a structure. The basic idea of the progressive collapse theory, as described in the two regulative documents (GSA, 2013; DOD, 2013), involves the analysis and redesign of the structural system following the loss of one of the primary load bearing elements of a structure. For the case of framed structures these elements are considered as the columns of the building.

Previous work by the authors has shown that the behavior of tall frames to component removal is almost always governed by loss-of-stability failures which can lead to the complete collapse of a building (Gerasimidis et al., 2016; Gerasimidis et al., 2016). However as shown in previous work (Sideri et al., 2016), the capacity of a tall building to an external blast event is higher when the lateral resisting system of the building is located at the perimeter of the structure. Although more analysis is necessary to validate this argument for diagrid structures, this is another major advantage of the diagrid structural system.

Another important point should be made here. The capacity of structural systems against unforeseen extreme events is mainly dependent on the alternate load path capacity of the system after the appearance of a localized damage. The application of an optimization process can lead to an outcome with limited alternate load paths and therefore have a decreasing effect on the capacity of the structure to extreme events. It is therefore considered highly important that optimization and robustness methods are coupled to produce an efficient and safe tall-building structural system.

5. Conclusions

This paper presents a simple optimization technique for the preliminary design of steel diagrid tall buildings. The technique was applied to three building geometries which

are common for diagrid structures. This work should be seen as preliminary and as a motivation for future work on the topics of diagrid optimization. In addition, the current paper discusses the topic of tall building robustness and optimization, two major fields in tall building design. Finally a future view of the diagrid structural system is provided at the end.

References

Moon, K.-S., Connor, J.J., and Fernandez, J.E. (2007). Diagrid structural systems for tall buildings: Characteristics and methodology for preliminary design. *The Structural Design of Tall and Special Buildings*, (16) 205-230.

Moon, K.-S. (2009). *Design and Construction of Steel Diagrid Structures.* NSCC, Malmo, Sweden, pp. 398-405.

Kim, Y.-J., Jung, I.-Y., Ju, Y.-K., Park, S.-J., and Kim, S.-D. (2010). Cyclic behavior of diagrid nodes with H-section braces. *ASCE Journal of Structural Engineering*, 136(9), 1111-1122.

Lee, D.-K., Starossek, U., and Shin, S.-M. (2010). Optimized topology extraction of steel-framed diagrid structure for tall buildings. *International Journal of Steel Structures,* 10(2), 157-164.

Xhang, C., Zhao, F., and Liu, Y. (2012). Diagrid tube structures composed of straight diagonals with gradually varying angles. *The Structural Design of Tall and Special Buildings*, (21) 283-295.

Mele, E., Toreno, M., Brandonisio, G., and de Luca, A. (2014a). Diagrid structures for tall buildings: Case studies and design considerations. *The Structural Design of Tall and Special Buildings*, (23) 124-145.

Montuori, G.M., Mele, E., Brandonisio, G., and de Luca, A. (2014b). Geometrical patterns for diagrid buildings: Exploring alternative design strategies from the structural point of view. *Engineering Structures*, (71) 112-127.

Zhao, F. and Zhang, C. (2015). Diagonal arrangements of diagrid tube structures for preliminary design. *The Structural Design of Tall and Special Buildings*, (24) 159-175.

Callow, J.A. (2001). *Development of software to optimize building design under lateral loading.* (Thesis). Princeton University.

Thornton, C.H., Joseph, L., and Scarangello, T. (1990). Optimization of tall structures for wind loading. *Journal of Wind Engineering and Industrial Aerodynamics*, (36) 235-244.

Applied Technology Council (ATC): http://windspeed.atcouncil.org/

Gerasimidis, S. and Sideri, T. (2016). A new partial distributed damage method for progressive collapse analysis of buildings. *Journal of Constructional Steel Research*, (119) pp. 233-245.

Gerasimidis, S. (2014). Analytical assessment of steel frames progressive collapse vulnerability to corner column loss. *Journal of Constructional Steel Research*, (95) pp. 1-9.

Sideri, J., Mullen, C., Gerasimidis, S., and Deodatis, G. (2016). *Progressive collapse vulnerability of 3D high rise steel buildings under external blast loading.* Proceedings Engineering Mechanics Institute Conference, Nashville, May.

DOD. (2013). *Unified Facilities Criteria (UFC), Design of buildings to resist progressive collapse.* Department of Defense, USA.

GSA. (2013). *Progressive collapse analysis and design guidelines for new federal buildings and major modernization projects.* General Services Administration, USA.

SIMULIA. (2015). *ABAQUS Theory Manual, Version 6.14.* Dassault Systemes.

Nejad, P. A. and Kim, J. (2011). *Beehive (Hexagrid), New Innovated Structural System for Tall Buildings.* Proceedings CTBUH Conference, Seoul.