Study on the Constitutive Relation of Structural Steel under Multi-axial Stress : Part 1 - uni-axial Cyclic Loading History

Kyungsoo Chung¹, Yuka Matsumoto², Satoshi Yamada³

¹ Graduate student, Interdisciplinary Graduate School of Science and Engineering, Tokyo Institute of Technology
² Research Associate, Department of Architecture, Yokohama National University
³ Associate Professor, Structural Engineering Research Center, Tokyo Institute of Technology

Abstract
The stress-strain relationship including the plastic deformation phase is essential information for structural design of buildings. However, lack of empirical data prevents the development of analysis model for multi-axial stress state.

With this background, the test setup was designed for the multi-axial loading. In this system, a cylindrical specimen is subjected to both axial force and torsion, and a uniform multi-axial stress can be generated in the cross-section. The loading equipment provides various loading paths, such as cyclic axial loading, cyclic torsion loading, alternate or simultaneous loading of axial force and torsion.

As the first stage of the whole project, this paper focuses on the uni-axial hysteretic model which is applicable to both axial stress and shear stress. For this purpose, axial loading tests and shear loading tests were performed, and the hysteretic behavior was investigated. The previously proposed model, which was derived from axial loading tests, was applied to these experiments, and the applicability to shear stress was confirmed.

Keywords: constitutive relation, multi-axial stress, uni-axial stress, hysteretic model

1. Introduction
For the ultimate seismic design of steel structure, it is important to estimate the hysteretic behavior and ultimate capacity of the member. The stress-strain relationship of material is essential information for this phase.

Among the subjects related to the material properties, stress-strain relation under multi-axial stress is a complicated problem. The isotropic hardening model and kinematic hardening model (Chen and Han, 1988) is generally used because of its simplicity, but the applicability is limited. Some researches proposed improved models, such as two-surface (Dafalias and Popov, 1976) and multi-surface models (Iwan, 1967). On the other hand, a more simplified prediction method is also required. However, lack of empirical data prevents the development of analysis model.

With this background, a series of researches was planned to perform the multi-axial loading experiments and to improve the stress-strain model for multi-axial stress. A loading equipment was designed for this project. In this system, a cylindrical specimen is subjected to both axial force and torsion, and a uniform multi-axial stress can be generated in the cross-section. The loading equipment provides various loading paths, such as cyclic axial loading, cyclic torsion loading, alternate or simultaneous loading of axial force and torsion.

This paper focuses on the uni-axial hysteretic model which is applicable to both axial stress and shear stress. For this purpose, axial loading tests and shear loading tests were performed, and the hysteretic behavior was investigated. Akiyama’s model (Akiyama and Takahashi, 1995; Yamada et al., 2002) was applied to these experiments, and the applicability to axial and shear stress was investigated.

2. Specimens and Experimental Procedures
2-1. Specimens and Material Properties
Two different structural steels, SS400 and SM490A, were chosen in this study. The material properties obtained by coupon tests are listed in Table 1. The measured yield stress of the ‘SM490A’, which was purchased as SM490A in market, didn’t satisfy the requirement for SM490A.

The specimen has cylindrical shape as shown in Fig.1. The axial force generates uniform axial stress and the torsion on its axis generates uniform shear stress in the cross-section area. The outer diameter and inner diameter of section area are 14mm and 11mm, respectively. The tube wall is thick enough to prevent the local buckling.
2-2. Experimental Procedures

Test setup is shown in Fig. 2. The loading equipment is composed of a reaction frame, an oil jack, a universal joint, a loading disk and a load cell. One end of the specimen was fixed to the reaction frame through the load cell, and the other end was fixed to the loading disk, which can rotate on its axis and slide in longitudinal direction of the setup. The loading disk was connected to the oil jack through the universal joint. The specimens were subjected to the axial force by the oil jack and the torsion by the rotation of the loading disk. The loading equipment can provide various loading paths, such as cyclic axial loading, cyclic torsion loading, alternate or simultaneous loading of axial force and torsion. As mentioned earlier, axial cyclic loading and shear cyclic loading were investigated in this paper. The loading types and loading sequences for all specimens are listed in Table 2. Tests were controlled by applying specified strain amplitude and performed at room temperature. The loading speed was quasi-static.

The load cell in Fig. 2 provided the axial load and the twisting moment separately. The axial and shear strain were measured by rectangular rosette strain gages and torque strain gages attached on the outer surface of the specimen.

2-3. Definition of Stress and Strain

The uniform stress distribution is assumed on the cross section. The nominal axial stress \( \sigma_n \) and shear stress \( \tau \) are calculated as follows.

\[
\sigma_n = \frac{P}{A} \quad \text{(1)}
\]

\[
\tau = \frac{M_t}{(2 \cdot t \cdot A')} \quad \text{(2)}
\]

Where, \( P \) and \( M_t \) are the axial force and twisting moment, respectively; \( A, t, \) and \( A' \) are the cross-section area, the tube thickness, and the area of average radius, respectively.
The nominal axial strain $\varepsilon$ and shear strain $\gamma$ were measured by strain gauges. The true axial stress $\sigma$ and true axial strain $\varepsilon$ are obtained as follows, based on the assumption of plastic incompressibility:

$$\sigma = (1 + \varepsilon) \times \sigma_n$$  \hspace{1cm} (3)

$$\varepsilon = \log(1 + \varepsilon_n)$$  \hspace{1cm} (4)

This paper focuses on the hysteretic model which can be applied to both axial stress and shear stress. For this purpose, shear stress $\tau$ is converted into equivalent axial stress, which is equal to $\tau$ in terms of effective stress $\sigma_{eq}$. Similarly, shear strain $\gamma$ is converted into equivalent axial strain $\varepsilon_{eq}$ by means of effective strain $\varepsilon_{eq}$. These stress and strain are defined as follows:

$$\sigma_{eq} = \sqrt{\sigma^2 + 3\tau^2}$$  \hspace{1cm} (5)

$$\varepsilon_{eq} = \sqrt{\varepsilon^2 + \gamma^2 / 3}$$  \hspace{1cm} (6)

$$\sigma = \sqrt{3} \times \tau , \quad \sigma_{eq} = \sigma$$  \hspace{1cm} (7)

$$\varepsilon = \frac{\gamma}{\sqrt{3}} , \quad \varepsilon_{eq} = \varepsilon$$  \hspace{1cm} (8)

3. Experimental Results

3-1. The Point of Consideration

The stress-strain relation under uni-axial cyclic loading is divided into the skeleton curve, bauschinger part and elastically unloading part as shown in Fig.4. This decomposition is a useful method of describing the hysteretic behavior of structural steel. The hysteretic model by the decomposition method was proposed in previous researches (Akiyama and Takahashi, 1995; Yamada et al., 2002). This model is based on the following empirical knowledge.

(a) The shape of skeleton curve is similar to the stress-strain curve under monotonic loading.

(b) The softening due to bauschinger effect is observed in bauschinger part, and the stiffness reduction depends on the accumulative plastic strain $\Sigma \varepsilon_S$ in the preceding loading history.

The aim of this study is to confirm the applicability of this model. Therefore, $\sigma_{eq} - \varepsilon$ relation for each specimen was divided as shown in Fig.4, and the properties of skeleton curve and the ones of bauschinger part were examined.

3-2. Hysteretic Behavior for Cyclic Loading

(1) Skeleton Curve

The skeleton curves of all specimens are shown in Fig.5. The arrows in these figures indicate the results of monotonic tensile tests. The skeleton curves of cyclic axial tests and the ones of all torsion tests are similar to monotonic tension curve. Therefore, it is conceivable that the skeleton curves for axial stress and ones for shear stress can be obtained by monotonic tensile test.

(a) SS400  
(b) SM490A

Fig. 5. Skeleton Curve

(2) Bauschinger Part

As mentioned above, the previous researches reported that the stiffness reduction in bauschinger part depends on the accumulative plastic strain in the preceding loading history. The similar tendency was observed in this study. In order to investigate it quantitatively, the index $\Delta e_B, \Sigma e_S$ and $\alpha_B$ were defined as shown in Fig.4 and Fig.6 and the correlations were examined.

The $\Delta e_B$, which is plastic strain amplitude in each segment of bauschinger part, reflects the stiffness

Fig. 4. Definition of Skeleton, Bauschinger and Unloading Part.

Fig. 6. Modeling of Bauschinger Part.
reduction. The relation between $\Delta \varepsilon_B$ and $\Sigma \varepsilon_B$, which is accumulative plastic strain in the preceding skeleton curve, is given in Fig. 7. The $\Delta \varepsilon_B$ is proportional to the $\Sigma \varepsilon_B$, and the coefficient is approximately 1/3. In addition, the influence of stress type (axial or shear) or materials was not observed.

In this study, each segment of bauschinger part was modeled as bi-linear, which is equivalent to the original segment in the energy dissipation. The initial stiffness of the bi-linear model is equal to the elastic modulus. The initial modulus is equal to the elastic modulus. The initial limit stress to $e_o$ is obtained as the ratio of elastic limit stress to $\sigma_B$, which is maximum stress in preceding skeleton part (see Fig. 6). The relation between $e_B$ and $\Delta \varepsilon_B$ is given in Fig. 8. The $\alpha_B$ is unrelated to $\Delta \varepsilon_B$, and it is allowable to consider $\alpha_B$ as constant, approximately 2/3.

Therefore, the following rules can be applied to the $\varepsilon_B$-stress relation in order to predict the hysteretic behavior for arbitrary uniaxial loadings.

(a) The skeleton curve has the same shape as stress-strain curve in monotonic tensile test.

(b) Each segment of bauschinger part is modeled as bi-linear, in which the initial stiffness is equal to elastic modulus. The elastic limit and the second stiffness are determined by $\Delta \varepsilon_B$, $\alpha_B$ and $\Sigma \varepsilon_B$. Considering the test results, $\Delta \varepsilon_B$ and $\alpha_B$ are evaluated as follows.

\[
\Delta \varepsilon_B = \frac{1}{3} \sum \varepsilon_S \quad - (9)
\]

\[
\alpha_B = 2/3 \quad - (10)
\]

(c) The gradient in unloading part is equal to elastic modulus.

(d) When the steel is unloaded during the bauschinger part, the following bauschinger part is oriented toward the previous unloading point.

The prediction procedure is illustrated in Fig. 9 and Fig. 10.
4-2. Adequacy of Model

The hysteretic loop was predicted by applying the model to the strain history of each specimen. The predicted loop and experimental loop are shown in Fig. 11 (a). In this figure, solid lines and dots indicate predicted data and experimental data, respectively. Equally, the predicted skeleton curve is compared with experimental data in Fig. 11 (b). The analysis model adequately described the specimen’s behavior.

The strain energy dissipation for predicted loop was also calculated, and it is compared with experimental data in Fig. 12 (a). The accumulative equivalent plastic strain in the whole skeleton curve was calculated from the predicted loop. The predicted accumulative strain is compared with experimental data in Fig. 12 (b). The analysis model adequately predicted the accumulative strain and energy dissipation, and the error was less than 20%.

From these results, the applicability of the hysteretic model to axial and shear stress was confirmed.

Specimen: SS-A-1

Specimen: SS-A-3

Specimen: SS-S-1

Specimen: SM-S-2

Specimen: SM-S-3

Specimen: SM-A-4

Fig. 11. Comparison of Experiment and Model in Hysteretic Loop and Skeleton Curve

(a) Stress-Strain Loops (b) Skeleton Curves

Fig. 11. Comparison of Experiment and Model in Hysteretic Loop and Skeleton Curve (Continue)

(a) Accumulated Energy Absorption (b) Accumulated Skeleton Strain

Fig. 12. Comparison of Experiment and Model in Accumulated Energy and Skeleton Strain
5. Conclusions

A series of researches was planned to perform the multi-axial loading experiments and to improve the stress-strain model for multi-axial stress. As the first stage of the project, the axial loading tests and shear loading tests were reported in this paper. The hysteretic behavior under shear stress was similar to one under axial stress in terms of equivalent stress and equivalent strain. The previously proposed model, which was derived from axial loading tests, was applied to these experiments, and it was confirmed that the hysteretic model is applicable to both axial stress and shear stress.

References