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Authors: Hee-Jung Ham, Assistant Professor, Kangwon National University
         Young-Moon Kim, Professor, Chonbuk National University
         Mi-Hwa Lee, Graduate Student, Kangwon National University

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Proper Orthogonal Decomposition Analysis of Dynamic Wind Pressures Acting on a Tall Chimney Model

Hee J. Ham\textsuperscript{1}, Young Moon Kim\textsuperscript{2}, Mi-Hwa Lee\textsuperscript{3}

\textsuperscript{1} Assistant Professor, Kangwon National University
\textsuperscript{2} Professor, Chonbuk National University
\textsuperscript{3} Former Graduate Student, Kangwon National University

Abstract

The wind-induced dynamic building pressures fluctuate irregularly according to time and space, and it is not easy to understand the complex behaviors of the building pressures by using conventional analysis tools such as statistical data reduction procedures (e.g., low and high moments), Fourier transform technique, etc. In this study, the proper orthogonal decomposition (POD) technique is applied to wind pressures acting on a tall chimney model, and the hidden dominant structures of dynamic building pressures are found and interpreted. From this study, the following results are obtained: the along-wind and across-wind forces are dominated by a few POD modes, and the base forces can be reconstructed by these POD modes with minor errors. In the paper, the physical meanings of dominant symmetric and asymmetric modes for the chimney pressures are also presented. The POD technique can compress complex natures of wind pressure field only by a few dominant modes and can interpret spatio-temporal characteristics of wind pressures by a novel way while existing statistical methods do not have such benefits.

Keywords: POD, Tall chimney, Dynamic pressure, Spatio-temporal characteristics

1. Introduction

The aerodynamic loading is random in nature and it has been investigated using statistical methods and elements of the random field theory. Spatio-temporal characteristics of the loading are of particular importance, especially when the aerodynamic loading and the dynamic response of structures or their components are considered. Two-point measurements are employed conventionally, and the space-time characteristics are evaluated either in time or frequency domain, employing either spatial correlation or the power cross-spectra.

It has been recognized in turbulence research that large scale temporal fluctuations of the aerodynamic loading are associated with flow events of relatively large spatial scales and low irregular frequency of occurrence. The notion of organized, large coherent structures has been developed and accepted in the wind engineering research community. At the same time it has been recognized that traditional statistical techniques do not provide adequate tools for analysis of such structures.

Application of the proper orthogonal decomposition (POD) was originally proposed by Lumley (1967) to extract statistically significant structures associated with turbulent flow fields. This technique was successfully employed in analysis of turbulent boundary-layer flows, Lu and Smith (1991), Moin and Moser (1989) and other studies. Both experimental and numerical data were employed in these studies.

Holmes (1981) proposed a covariance method which, as shown by Bienkiewicz et al. (1993 and 1995), is a special case of the POD technique. The covariance vector technique was implemented by Holmes in one-dimensional applications. The spatial correlation of the strip of the surface pressure was employed to extract the eigenvalues and eigenfunctions. The significance of these quantities has been evaluated, and the similarity between the first two modes and the pressure characteristics has been noted. Application of the POD method to simplify the description of wind loading and wind-induced structural response was discussed by Davenport (1995) and Carassale et al. (2001). Tamura et al. (1995) used the POD technique to analyze the correlation between the upstream wind characteristics and the building pressure field. This analysis indicates that an upstream wind monitored at the building roof height may not necessarily be the best choice as the reference wind speed for the approach wind.

This paper briefly outlines the POD technique, and the application of the POD method is illustrated through
decomposition analysis of wind-tunnel pressure, distributed in wall regions of a high-rise chimney model.

2. Background

The main objective of the proper orthogonal decomposition technique is to establish a deterministic function \( \phi(x,y) \) which is the best correlated with all the elements of the ensemble of a random field. Given a random signal \( p(x,y,t) \), the maximum of the projection of \( p(x,y,t) \) on the function \( \phi(x,y) \) is sought. An integral form of this operation is as follow:

\[
\int p(x,y,t) \phi(x,y) dx dy = \max \tag{1}
\]

and its normalized counterpart is:

\[
\frac{\int p(x,y,t) \phi(x,y) dx dy}{\sqrt{\int \phi^2(x,y) dx dy}} = \max. \tag{2}
\]

Since the building pressure is random and it takes on both positive and negative values, the ensemble of the square of Eq. (1) is considered in the POD analysis and its maximum is sought:

\[
< \left[ \int p(x,y) \phi(x,y) dx dy \right] \left[ \int p(x',y',t) \phi(x',y') dx' dy' \right] > \\
\int \phi^2(x,y) dx dy = \lambda > 0. \tag{3}
\]

This leads to an eigenvalue problem:

\[
\int R_p(x,y,x',y') \phi(x',y') dx' dy' = \lambda \phi(x,y) \tag{4}
\]

where \( R_p(x,y,x',y') \) is the space covariance of the signal. This integration is performed numerically when discrete values of the signal are specified. In this simplest case, in which the signal is specified at uniformly spaced locations and the rectangular rule is employed in the integration, Eq. (4) can be replaced by

\[
[R_p] \phi = \lambda \phi \tag{5}
\]

where \([R_p]\) is the covariance matrix and \( \phi \) is the eigenfunction.

Once the space covariance matrix of pressure is known, the eigenvalue problem Eq. (4) is solved and the eigenvalues \( \lambda_n \) and the eigenfunction \( \phi(x,y) \) are computed. The eigenfunctions are used as the base function in a series expansion of the signal

\[
p(x,y,t) = \sum a_n(t) \phi_n(x,y) \tag{6}
\]

where the principal coordinates are

\[
< a_n(t) > = \frac{\int p(x,y,t) \phi_n(x,y) dx dy}{\phi_n^2(x,y) dx dy}. \tag{7}
\]

In Eq. (7), the principal coordinates are

\[
\int \phi_n(x,y) \phi_m(x,y) dx dy = \delta_{nm}. \tag{8}
\]

In Eq. (7), the eigenfunctions can be assumed to be orthonormal,

\[
< a_n(t)a_m(t) > = \sum \lambda_n \delta_{nm}(x,y). \tag{9}
\]

Where \( \delta_{nm} \) is the Kronecker delta.

3. Data acquisition

External building pressure under suburban flow condition was acquired nearly simultaneously for a tall chimney model at a boundary layer wind-tunnel of Daewoo Engineering and Construction Ltd. in Korea. An electrical pressure scanning system was employed for the pressure measurement. Fig. 1 shows 120 numbers of pressure tap locations on the model.

![Fig. 1. Experimental setups and pressure tap locations](image-url)
4. POD analysis

The mean and the root mean square of the pressure are shown in Fig. 2. Total fluctuating pressure including the pressure mean was employed in the POD analysis. Mean value distribution is slightly departure from the expected symmetry. This departure was not further investigated, nor were the data corrected. The first six eigenvectors are shown in Fig. 3. In the figures, left vertical axis represents model elevation while the right vertical axis stands for normalized building elevation (z/H).

The power spectral densities (PSD) of the generalized wind forces for along-wind and across-wind directions obtained from the chimney pressure are shown in Fig. 4. Fig. 5 shows the power spectral densities for each principal coordinate.

Fig. 2. Mean and RMS pressure distributions

Fig. 3. First six eigenfunctions

Fig. 4. PSD of wind forces

Fig. 5. PSD of the first six principal coordinates

...Fig. 6 represents the vertical distribution of the RMS wind force obtained by each eigenfunction and its principal coordinate.

The characteristics of the first four POD modes are as follows:

The 1\textsuperscript{st} eigenfunction is symmetric as shown in Fig 3. The shape is quite similar to the mean pressure distribution as shown in Fig. 2. This observation agrees with the results of Bienkiewicz et al. (1995). The power spectral density of the 1\textsuperscript{st} mode principal coordinate in Fig. 5 is similar to the along-wind force as shown in Fig. 4. The RMS distribution of along-wind force obtained from the 1\textsuperscript{st} mode in Fig. 6(a) is similar to that of original, and is dominant in magnitude. The energy contribution of the 1\textsuperscript{st} mode to the RMS distribution of across-wind force is negligible as shown in Fig. 6(b).

The 2\textsuperscript{nd} eigenfunction is asymmetric as shown in Fig 3. There are positive and negative maxima at 0.6
H of the side walls, and this reflects strong vortex shedding formations as Lee (1975), Kareem and Cermak (1984), and Kikucki et al. (1997) pointed out. The power spectral density of the 2nd mode principal coordinate in Fig. 5 is quite similar to the across-wind force as shown in Fig. 4. It can be observed that the reduced frequencies at peak values are same. The RMS distribution of across-wind force obtained from the 2nd mode in Fig. 6(b) is similar to that of original, and is dominant in magnitude. The energy contribution of the 2nd mode to the RMS distribution of along-wind force is negligible as shown in Fig. 6(a).

The 3rd eigenfunction is asymmetric. There are high value regions at heights 0.8H and 0.2H on the side walls with opposite signs. The RMS across-wind force derived from the 3rd mode is also large at height 0.8H and 0.2H. The power spectral density of the 3rd mode principal coordinate in Fig. 5 is quite similar to the across-wind force with same frequency of the peak values as shown in Fig. 4. This suggests that additional vortex shedding occurs at the high value regions of the 3rd eigenfunction in addition to the 2nd mode.

The 4th eigenfunction is symmetric nature as shown in Fig 3. There are high value regions at heights 0.9H and 0.2H on the windward wall with opposite signs. The power spectral density of the 1st mode principal coordinate in Fig. 5 is similar to the along-wind force as shown in Fig. 4, when the reduced frequency nz/U is higher than 0.01. The RMS distribution of along-wind force obtained from the 4th mode in Fig. 6(a). The energy contribution of the 4th mode to the RMS distribution of across-wind force is negligible as shown in Fig. 6(b).

Further evidence of the significance of the selected few modes is illustrated in Fig. 7 and Fig. 8, which show the original and reconstructed wind forces. Fig. 7 shows a convergence of the reconstructed RMS forces. The 1st, 4th, and 6th POD modes and the 2nd, 3rd, and 5th POD modes are used for along-wind and across-wind RMS forces, respectively. It can be seen that the reconstructions with the selected few modes represent overall behaviors of original RMS forces well.
Fig. 8 depicts a convergence of the reconstructed along and across base forces for the selected POD modes. A portion of the force records in the vicinity of the largest peak is shown in the figures. For the reconstruction, the 1st, 4th, 6th, and 10th POD modes are used for along-wind direction while the 2nd, 3rd, 5th, and 7th POD modes are used for across-wind direction. It can be seen that the agreement between the original and reconstructed force records is very good even when only the selected few modes are used.

5. Conclusions

The presented data show that the results of the proper orthogonal decomposition of the multi-channel wall pressure exhibit properties similar to those reported by other researchers. The POD analysis shows that the shape of the first mode is similar to that of the mean pressure. The shape of second eigenvector has an asymmetric nature. This result reflects strong vortex shedding formations on the tall chimney. The first POD mode makes the most significant contributions to the along-wind base force while the second POD mode contributes to the across-wind base force significantly.

This study confirms that the POD method is very useful to compress complex natures of wind forces only by a few dominant modes and can interpret their spatio-temporal characteristics by a novel way.

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